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HAZAN: A FORTRAN PROGRAM TO EVALUATE SEISMIC-HAZARD PARAMETERS USING GUMBEL'S THEORY OF EXTREME VALUE STATISTICS

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Abstract—A FORTRAN IV computer program for seismic Hazard Analysis is presented and illustrated by an example. It evaluates the parameters of Gumbel's first and third type asymptotic distributions of extreme values and in the latter situation it is based on the nonlinear least-squares method developed by Marquardt. The application of this method is developed here so that the uncertainty on each parameter of Gumbel's third distribution is estimated and the complete covariance matrix is obtained, in recognition of the importance of assigning uncertainties to all parameters used for seismic hazard assessment. The primary data required is a chronological catalogue of earthquake magnitudes, however, earthquake magnitude or a related value of earthquake caused ground acceleration, velocity, or displacement may be the quantity used in the ensuing analysis to characterize the seismic hazard.

Key Words: Extreme values, Seismic hazard, Prediction, Uncertainties.

INTRODUCTION

The probabilistic nature of forecasting the size of earthquake occurrences such as the expected magnitude, ground acceleration, velocity, or displacement, leads seismologists to use statistical models which have proved to be reliable decision-making tools elsewhere. Several statistical models have been applied to the analysis of earthquake-occurrence sequences with differing degrees of success. Many of the results obtained are unsatisfactory because of undetected incompleteness in the data sets analyzed or because even rudimentary studies into the inherent uncertainties associated with the parameters to describe statistically the earthquake population are deemed to be of little consequence, or worse, omitted (Burton, 1979). However, the knowledge of the precision with which the parameters are calculated is an important factor in examining the stability of the system and accordingly evaluating the results before inherent economic implications can be estimated.

Generally, the statistical treatment used falls into one of two categories: (a) "whole-process" methods using the whole-data set such as the Gutenberg–Richter cumulative frequency law (Gutenberg and Richter, 1944), and (b) "part-process" methods using part of the data set such as the annual extremes. Advantages and limitations of both methodologies are discussed elsewhere (for example see Lomnitz, 1974; Yegulalp and Kuo, 1974; Makropoulos, 1978; Burton, 1978, 1979; Burton, Main, and Long, 1983). It has been shown that in terms of evaluating the hazard associated with large damaging earthquakes the models of the second category may be more appropriate given the available data and earthquake catalogs. Such a model is the one described by Gumbel (1958). It has attracted widespread interest and has been adapted by geoscientists for hydrological computations, climatic evaluations (Jenkinson, 1955), as well as for seismic-hazard determination (also see Epstein and Lomnitz, 1966; Schenkova and Karnik, 1976; Burton and others, 1984).

The program described here has gone through several variations, and it was developed for our previous work on estimating hazard parameters using Gumbel's statistical model of extreme values, in recognition of the importance of having uncertainties for all the calculated hazard parameters. A user may well wish to make modifications to this program to suit his needs and in some respects some parts of this program merely are illustrative—ready to be adapted. Some of the more obvious modifications possible are mentioned later, including the scope to adapt the program to particular regional needs. The program given here includes some general options.

BRIEF DESCRIPTION OF THE MODEL

Although details of the theory and many references to the basic original contributions are presented in the previous papers, it will be useful to summarize the main points here.

Irrespective of the parent distribution, the distribution from which the extremes are sampled must take one of three forms (Davis, 1970). We will label these the first, second, and third type asymptotic distributions of Gumbel, or Gumbel I, II, and III respectively. The three asymptotes are:

$$1)\ G^{I}(x) = \exp(-\exp(-\alpha(x-u))), \quad \alpha > 0 \quad (1)$$

$$2)\ G^{II}(x) = \exp(-((u-\gamma)/(x-\gamma))^{k}), \quad k > 0, \quad x \geq \gamma, \quad u > \gamma \geq 0 \quad (2)$$

$$3)\ G^{III}(x) = \exp(-((\omega - x)/(\omega - u))^{k}), \quad k > 0, \quad x \leq \omega, \quad u < \omega, \quad (3)$$

where in each situation G(x) is the probability that the variable x is an annual extreme, that is, G(x) is the annual probability that x is not exceeded. In all three situations the parameter u is a characteristic value of the variable x and has the probability $G(u) = 1/e$ of being an annual extreme.

The first type in Equation (1) holds for initial whole-process distributions unlimited in both directions of the variable x; in addition to u it has the second parameter α. The second type, Equation (2), arises when the initial distribution is bounded below at $x \geq \gamma$. For the purpose of maximum earthquakes, Gumbel II is ruled out, and the third type, Equation (3), results when the initial whole-process distribution is bounded towards the right at $x \leq \omega$. Both Gumbel II and III have a third parameter k which may be seen as representing the curvature of the distribution, and as k increases both Gumbel II and III reduce to the linear type Gumbel I distribution. The third type is used for maximum magnitudes where physically realistic curvature at higher magnitudes, which may be obvious in the data, is taken into account; whereas the first type is recommended for use with peak ground acceleration, velocity, or displacement calculations. For simplicity, $G^{I,II,III}(x)$ will be replaced by P(x), or simply P, and Gumbel II will not be considered for Hazard Analysis.

CURVE-FITTING TECHNIQUE AND PARAMETER ESTIMATION

It is easy relatively to estimate the parameters of the first type asymptote, defined in Equation (1), by simple linear least-squares (Bevington, 1969). After introducing the 'reduced variable' y as

$$y = -\ln(-\ln P(x)). \quad (4)$$

Equation (1) becomes a straight line of the form

$$y = \alpha(x - u) \quad \text{or} \quad x = u - y/\alpha. \quad (5)$$

A subroutine termed LINFIT in the program (Appendix 1) calculates the parameters and the associated uncertainties allowing weight for each individual earthquake magnitude or ground acceleration, etc. to be taken into account. However, because of nonlinearity in the parameters ω, u, and k of Gumbel III, Equation (3), the conventional least-squares method cannot be applied directly to estimate them. The method to be used here is nonlinear least squares based on the technique outlined by Levenberg (1944), developed by Marquardt (1963), and discussed and programmed by Bevington (1969). The application of the method is developed further here so that the uncertainty on each parameter of the distribution is estimated and the complete covariance or error matrix is obtained. The importance of the covariance matrix has been established in all our previous work when the Gumbel III asymptote is used for prediction of earthquake occurrence at known levels.

Equation (3) may be transposed to give x as

$$x = \omega - (\omega - u)(-\ln(P(x)))^{\lambda}, \quad (6)$$

where $\lambda = 1/k$. The usual procedure for fitting a nonlinear function $y(x)$ is to expand $y(x)$ linearly in a Taylor series function of parameters p_j and then perform linear least squares to obtain optimum values for perturbations δp_j to the initial trial values of p_j. Gumbel III has three parameters ω, u and λ for p_j and so the expansion is:

$$y(x) = y_o(x) + \Sigma\left(\frac{\partial y_o(x)}{\partial p_j}\,\delta p_j\right), \quad j = 1, 2, 3, \quad (7)$$

and if this function is fitted to the n observables y_i (typically the y_i will be n observed annual extreme earthquake magnitudes) then the goodness-of-fit may be measured by χ^2 with

$$\chi^2 = \Sigma\left(\frac{1}{\sigma_i^2}(y_i - y(x_i))\right)^2, \quad i = 1 \cdots n, \quad (8)$$

where σ_i is the standard deviation associated with each datum. χ^2 is minimized with respect to each parameter leading to the matrix equation:

$$\mathbf{B} = \delta p \mathbf{A}. \quad (9)$$

Elements of **A** and **B** are given by:

$$\mathbf{A}_{jk} = \Sigma\left(\frac{1}{\sigma_i^2}\frac{\partial y_o(x_i)}{\partial p_j}\frac{\partial y_o(x_i)}{\partial p_k}\right) \quad (10)$$

and

$$\mathbf{B}_k = \Sigma\left(\frac{1}{\sigma_i^2}(y_i - y_o(x_i))\frac{\partial y_o(x_i)}{\partial p_k}\right), \quad (11)$$

where the solution of Equation (9) is given by

$$\delta p = \mathbf{B}\mathbf{A}^{-1} = \mathbf{B}\epsilon, \quad (12)$$

and ϵ is the symmetrical covariance or error matrix. Using Equation (6) as the fitting function requires the three parameters p_1, p_2, p_3, that is ω, u, and λ, respectively. The covariance matrix ϵ of Equation (12) is explicitly:

$$\epsilon_{ij} = \begin{bmatrix} \sigma_\omega^2 & \sigma_{u\omega}^2 & \sigma_{\lambda\omega}^2 \\ \sigma_{\omega u}^2 & \sigma_u^2 & \sigma_{\lambda u}^2 \\ \sigma_{\omega\lambda}^2 & \sigma_{u\lambda}^2 & \sigma_\lambda^2 \end{bmatrix}, \qquad (13)$$

and the parameter uncertainties are obtained from the diagonal elements. Additionally, the off-diagonal elements give evidence of dependence between the parameters. Equation (7) includes the partial derivatives with respect to each parameter, which from Equation (6) are:

$$\frac{\partial x}{\partial \omega} = 1 - (-\ln \mathrm{P})^\lambda,$$

$$\frac{\partial x}{\partial u} = (-\ln \mathrm{P})^\lambda,$$

$$\frac{\partial x}{\partial \lambda} = (\omega - u)(-\ln \mathrm{P})^\lambda(\ln(-\ln \mathrm{P})). \qquad (14)$$

Marquardt (1963) suggests an algorithm for solving Equation (9) and other similar equations. This relies on increasing the diagonal elements of matrix **A** by a factor η. When η is large the off-diagonal terms are trivial, and the diagonal terms dominate. Equation (9) then degenerates into separate equations:

$$\mathbf{B}_j = \eta \delta p_j \mathbf{A}_{jj}. \qquad (15)$$

When η is small the solution reduces to that obtained using the complete linearized matrix of equation (9), which then becomes:

$$\mathbf{B} = \delta p \mathbf{C},$$

with

$$\mathbf{C}_{jk} = \mathbf{A}_{jk}(1 + \eta) \quad j = k,$$

$$\mathbf{C}_{jk} = \mathbf{A}_{jk} \quad j \neq k. \qquad (16)$$

The overall procedure is efficient if η is adjusted carefully during the iterations and Marquardt suggests a suitable iteration scheme. η is decreased so that eventually the final iterations approach as nearly as possible to the analytical linearized solution dictated by Equation (9). There are several options for testing convergence. The number of iterations may be fixed, although this gives no information about the degree of convergence achieved. In practice, values of ω, u, and λ are accepted here when the reduced χ^2 generated by successive iterations differs by less than 0.001. Elements of the covariance matrix in Equation (13) are calculated when η is a small value.

All the computations are performed by calling the subroutine CURFIT with the exception that the main program decides, on the basis of χ^2 determined, if the values of ω, u, and λ are acceptable or if another iteration is needed.

COMPUTATIONAL PROCEDURE AND HAZARD VALUES

The main program HAZAN initiates the procedure by creating a mesh of grid points in latitude and longitude with the given spacing STEP° for which the analysis is to be performed. Next, for each grid point and for a circular area around it specified by the radial distance SIZE°, the program extracts all earthquakes from the catalog which have occurred in the area in excess of the selected magnitude threshold during the required time interval. The chronologically ordered earthquake catalog will contain earthquake dates and epicentral parameters in the form of latitude, longitude, and magnitude (and preferably focal depth km if ground motion hazard analysis is to be performed). So, it should be noted that a simple cell-like sort is performed first to obtain earthquakes with epicentral positions within ± SIZE° latitude and longitude of the grid point [in fact SIZE° is corrected for latitude by the factor 1/cos (latitude)] and then these earthquakes are screened to ensure they are within SIZE km (assuming 1° equivalent to 111.1 km) of the grid point. The third step is to create a subset of data containing the maximum value per annum, in each cell, for the hazard variable selected.

The hazard variable x may be selected to be earthquake magnitude M, ground acceleration a cm/s^2, ground velocity v cm/s, or displacement d cm. If the hazard variable to be analyzed is the maximum earthquake magnitude then this procedure is a straightforward comparison. However, if a maximum acceleration, velocity, or displacement hazard analysis has been selected, then a corresponding attenuation law is applied first to each of the earthquakes in a cell to evaluate the corresponding ground motion at the grid point. Such attenuation laws are generally of the form

$$Y = b_1 e^{b_2 M}(r + k)^{-b_3} \qquad (17)$$

where Y is the ground motion value at the site for an earthquake magnitude M at focal distance r km. The correction k accounts for finite focal volume and b_1, b_2, and b_3 are constants for the selected attenuation law. The attenuation laws included in HAZAN are, for peak ground acceleration (Makropoulos, 1978; Makropoulos and Burton, 1985):

$$a = 2164\ e^{0.7M}(r + 20)^{-1.8}\ \mathrm{cm/s^2}, \qquad (18)$$

for ground velocity (Orphal and Lahoud, 1974):

$$v = 0.726 \cdot 10^{0.52M} r^{-1.34}\ \mathrm{cm/s}, \qquad (19)$$

and for ground displacement (Orphal and Lahoud, 1974):

$$d = 0.0471 \cdot 10^{0.57M} r^{-1.18}\ \mathrm{cm}. \qquad (20)$$

When a ground-motion value has been calculated, at the grid point, for an earthquake, then, the previous third step may be carried out as in the simpler situation of maximum magnitudes. At completion of this stage the main program of HAZAN has sorted the earthquake catalog and retained, for each cell, the annual extremes of the selected hazard variable.

The next major stage processes the individual cells of data. First, the n observed annual extremes x_i ($i = 1 \cdots n$) in a cell are ranked (subroutine RANK) into ascending size and an observed probability of being an annual extreme assigned to each using Gringorten's (1963) formula:

$$P(x_i) = (i - 0.44)/(n + 0.12). \qquad (21)$$

Prior to fitting the selected extreme value distribution to the data, it is possible to assign individual standard deviations σ_i to each observed extreme x_i. The program in Appendix 1 includes two possibilities, mainly for illustrative purposes, and these are: weighting extreme magnitudes according to a limited set of four magnitude ranges; weighting each extreme acceleration in proportion to the value of the acceleration (the latter after McGuire, 1974). Alternatives may be inserted easily into the program.

By this stage the main program has performed the preparatory work needed for the subroutines LINFIT or CURFIT to calculate either the parameters α and u of Gumbel I in Equation (5), using the simple linear least-squares method, or the parameters ω, u, and λ and their uncertainties by applying the nonlinear least-squares method by linearizing the fitting function given by Gumbel III in Equation (6), as described previously. No matter which option—Gumbel I or Gumbel III—the program prints out the distribution parameters and their uncertainties. In the situation of Gumbel III the off-diagonal covariances of the error matrix ϵ, Equation (13), also are printed. Some of the possible output is described later.

Finally, using the parameters of the selected distribution the main program computes hazard parameters such as the annual mode, T-years mode, and the maximum expected magnitude or acceleration etc., of not being exceeded in T-years at a stated probability level. For example, if Gumbel I has been selected, then the hazard value representing the T-year mode with probability P of not being exceeded, $x(T, P)$, is given by:

$$x(T, P) = u - \frac{\ln(-\ln P)}{\alpha} + \frac{\ln T}{\alpha}. \qquad (22)$$

Note that HAZAN calculates $x(T, P)$ using Gumbel I for values of T fixed at 1, 25, 50, 100, and 200 years whereas the probability level P is data input to the program (in practice the probability of exceeding, 1 − P, is input). The procedure is slightly different with Gumbel III, mainly because it is skew. The T-year modal value, $x(T)$, is given by:

$$x(T) = \omega - (\omega - u)\left(\frac{1-\lambda}{T}\right)^{\lambda}, \qquad (23)$$

and the hazard value with probability P of not being exceeded in T-years, $x(T, P)$, is given by:

$$a(T, P) = \omega - (\omega - u)\left(-\frac{\ln P}{T}\right)^{\lambda}. \qquad (24)$$

This version of HAZAN calculates $x(T)$ for T fixed at 1 and 75 years, that is the annual and 75-year modal extreme values. It also calculates $x(T, P)$ giving the values with a 90% probability of being an annual extreme and that with a 90% probability of being a 75-year extreme (90% probability of not being exceeded in 75 years). Other values easily could be calculated by HAZAN to characterize the seismic hazard.

Modifications

There are many possible modifications to this program, and it is intended that the user will adapt it to his needs. Modifications both may be seismological and computational in nature. For instance, annual extremes are not appropriate always, particularly in areas of low seismicity, and two-, three-, or more-yearly extremes may be required. A user also may wish to insert an attenuation law appropriate to his own region, or to use macroseismic intensity values rather than magnitude on ground acceleration, etc. It also is an easy matter to modify the hazard calculations, for example to use Equation (23) to calculate the 100-year mode rather than the 75-year mode given by the present program. Computational changes also may be made: the present program allows for a 5 × 5 grid of latitude and longitude, which may be more-or-less than the storage required by another user. HAZAN is meant to be adaptable and to this end the input requirements have been kept relatively simple.

INPUT REQUIREMENTS AND AN EXAMPLE APPLICATION

The methods and program described here, or variants of it, have been applied to our previous studies (see references cited). However, for demonstration purposes, a selected hazard analysis for the area around Corinth, Greece (37.95°N, 22.92°E), will be presented as an example application in conjunction with a step-by-step guide to using HAZAN. The input requirements to HAZAN are three (or four) input data cards and an earthquake catalog.

Input

HAZAN uses two data-set reference numbers on input, these being stream 5 and stream 3, the latter for the earthquake catalog. In what follows the heavy brackets contain specimen input data for a Gumbel III hazard analysis of magnitude occurrence in the Corinth area.

(1) DATA STREAM 5: selection of area, grid, and methodology.

FIRST CARD

FORMAT(6F6.2) e.g. (37.95, 37.95, 22.42, 23.42, 0.5, 1.)

- BOLA: Bottom Latitude of the whole Area.
- TOLA: Top Latitude of the whole Area.
- LFLO: Left Longitude of the whole Area.
- RTLO: Right Longitude of the whole Area.
- STEP: Step or shift in Latitude and Longitude between grid points.
- SIZE: Radius, in degrees, of the area from each grid point which constitutes a cell for hazard analysis. A degree is taken to be 111.1 km.

SECOND CARD

FORMAT(7I5) e.g., (4, 1, 1900, 1981, 1, 0, 2)

- IDENT: This selects if earthquake magnitude or ground acceleration, velocity, or displacement will be used to characterize the seismic hazard.
  - = 1 For Maximum Acceleration Distribution.
  - = 2 For Maximum Velocity Distribution.
  - = 3 For Maximum Displacement Distribution.
  - = 4 For Maximum Magnitude Distribution.
- MODE: This selects how the observed annual extremes x_i will be weighted.
  - = −1 Weight to be $1/x_i$.
  - = 0 Equals weights on all x_i data.
  - = 1 Weight to be 1/SIGMA(I), where SIGMA(I) are written specifically into HAZAN (there are two examples in the present program for illustrative purposes).
- MINT: Starting year of the period of investigation.
- MAXT: Final year of the period of investigation.
- LIST1: These specify printout options and either or
- LIST2: both may be set to 0 or 1. Typically both will be set to 0. LIST2 = 1 gives a more extensive printout including the ranked extremes in a cell and LIST1 = 1 should be used only if the user wants vast output detailing each earthquake in each cell, etc.
- INT: Selects the first or third asymptote of Gumbel.
  - = 1 Selects the first asymptote, see Equation (5).
  - = 2 Selects the third asymptote, see Equation (6).

THIRD CARD

FORMAT(2F6.2) e.g. (4.0, 0.30)

- EMMIN: Minimum Magnitude threshold to be considered
- PROB: Probability Level at which the hazard value is expected to be exceeded in T-years, see Equation (22).
  This is required only when Gumbel I has been selected by setting INT = 1 on the second card.

FOURTH CARD

FORMAT(3F7.3) e.g. (7.0, 4.5, .3)

Note that this card is required only if the third asymptote or Gumbel III has been selected by setting INT = 2 on card 2, otherwise omit this card.

- A1(1): A starting value for the parameter ω, see Equation (6).
- A1(2): A starting value for the parameter u, see Equation (6).
- A1(3): A starting value for the parameter λ, see Equation (6).

(2) DATA STREAM 3: the earthquake catalog.

This data stream contains the earthquake catalog, and it has as many cards as there are earthquakes in the catalog. The earthquake catalog must be in chronological order and contain for each earthquake at least the year, latitude, longitude, magnitude (and preferably depth, which is needed when ground motion rather than magnitude seismic hazard is calculated).

Appendix 2 lists some specimen earthquake catalog data which are suitable for test purposes only (many earthquakes have been eliminated leaving mainly the annual extremes of magnitude). These data have been extracted for the Corinth area from the earthquake catalog of Makropoulos and Burton (1981), supplemented for the years 1979–1981 by the Bulletins of the International Seismological Centre. The FORMAT is labeled as statement number 9060 in HAZAN and maybe modified to suit the user's earthquake catalog.

FIRST CARD

FORMAT (1X, I4, 4A4, F4.1, 2F8.2, I5, 9X, F3.1) e.g. see Appendix 2.

- IYEAR: The year of this earthquake.
- TITLE: These are not needed by
- O3: HAZAN, however the corresponding 20 columns of the earthquake catalog used (see

Appendix 2) contain the month, day, hour, minute, and second of the earthquake.

EN: The latitude of this earthquake

BO: The longitude of this earthquake

IDEPTH: The focal depth km of this earthquake. Needed when ground motion seismic hazard is to be evaluated.

EM: The magnitude of this earthquake

,
,
,
,

n'th CARD Last card of the earthquake catalog.

Output:

HAZAN may use two data set reference numbers on output, these being stream 6 and stream 7. If Gumbel III has been selected, then both streams are used; stream 7 containing a summary table of hazard values. With Gumbel I all output occurs in stream 6. Selected parts of the output from the program, using the Corinth data example (Appendix 2) as input on stream 3 with the data input on stream 5 set as indicated, are shown in Appendix 3. These results are mostly self explanatory and are presented for one cell analyzed only. LIST2 was set to 1 and so a preliminary table is printed showing the year index (K), and the year, the size (AMP) of the extreme value in that year. The next four columns of this table show the previous annual extremes now ranked into ascending size (RANKED), the reduced variate ($G(Y) = -\ln(-\ln \text{PROB})$), the observed probability of being an annual extreme (PROB) and the index of the ranked extremes (L). This is followed by the output giving the Gumbel III distribution parameters (which was selected in this example) and uncertainties and covariances extracted from the elements ϵ_{ij} in ϵ of Equation (13). In this output COV is the covariance among the three parameters ω, u, and λ. The first number shows the row and the second the column of the elements of matrix (13). As far as the summary hazard values in stream 7 are concerned, the meaning of the abbreviations of the table headings is as follows:

GEOGR.COORD: the geographical coordinates of each grid point.

AN. MODE: the most probable annual maximum magnitude.

90% NBE: the maximum magnitude with 90% probability of not being exceeded annually.

75Y MODE: the most probable maximum magnitude during the next 75 years.

75Y 90% NBE: the maximum magnitude with 90% probability of not being exceeded in the next 75 years.

MAXOB: the maximum observed magnitude in the area under study.

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APPENDIX 1

Annotated listing of FORTRAN computer program HAZAN

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C           H  A  Z  A  N     P  R  O  G  R  A  M
C           ------------------------------------
C
C       FOR SEISMIC HAZARD ANALYSIS:
C       THIS PROGRAM COMPUTES THE PARAMETERS OF GUMBEL'S FIRST AND
C       THIRD TYPE ASYMPTOTIC DISTRIBUTIONS OF EXTREMES. UNCERTAINTIES
C       ON THE PARAMETERS ARE DETERMINED AND FOR GUMBEL'S THIRD TYPE
C       DISTRIBUTION THE ERROR MATRIX IS OBTAINED. USING GUMBEL PARAMETERS
C       HAZAN COMPUTES PREDICTION PARAMETERS SUCH AS: ANNUAL MODE, T-YEARS
C       MODE, MAXIMUM EXPECTED MAGNITUDE OR ACCELERATION OF NOT BEING
C       EXCEEDED IN T YEARS. THE CALCULATIONS MAY BE APPLIED TO ANNUAL
C       EXTREMES OF EARTHQUAKE MAGNITUDE OR TO RELATED VALUES OF GROUND
C       ACCELERATION, VELOCITY OR DISPLACEMENT.
C
C       THE PARAMETERS OF THE THIRD TYPE ASYMPTOTE, EQUATION (3) IN
C       THE MAIN TEXT, ARE COMPUTED USING MARQUARDT'S(1963) ALGORITHM,
C       BY LINEARISING THE FITTING FUNCTION:
C
C               M=W-(W-U)*((ALOG(P(M))))**L       (SEE EQ.(6))
C
C       STARTING WITH INITIAL TRIAL VALUES OF W,U,AND L=1/K,(OMEGA,U
C       AND LAMBDA) THE GOODNESS OF FIT TO THE N OBSERVATIONS IS MEASURED
C       BY THE REDUCED CHI-SQUARE WHICH IS MINIMISED WITH RESPECT TO
C       EACH PARAMETER, LEADING TO THE LINEAR MATRIX EQUATION :
C
C               BETA=DELTAA(I)*ALFA               (SEE EQ.(9))
C
C       THE UNCERTAINTIES ON W,U AND L ARE CALCULATED FROM:
C
C               DELTAA(I)=BETA*E                  (SEE EQ.(12))
C
C       WHERE E IS THE INVERSE MATRIX OF ALFA. E IS THE ERROR MATRIX
C       AND ITS ELEMENTS ARE THE VARIANCE AND COVARIANCE OF THE
C       PARAMETERS W, U AND L, SEE EQ.(13)
C
C       THE PARAMETERS OF THE FIRST TYPE ASYMPTOTE, EQ.(1), A AND U ARE
C       COMPUTED USING LINEAR LEAST-SQUARES METHOD WITH THE FITTING
C       FUNCTION:
C               M=U+(1/A)*Y                       (SEE EQ.(5))
C
C       WHERE:
C               Y=-ALOG(-ALOG(P(M)))              (SEE EQ.(4))
C
C       FOR DESCRIPTION OF THE INPUT DATA SEE MAIN TEXT
C       FOR DESCRIPTION OF THE OUTPUT STREAMS SEE MAIN TEXT
C
C
C       ---------------------------------------------------------------------
C
C
C       MAIN
C
        DIMENSION A(5,5,95),GY(95),YM(95),Y(95),RY(4),TITLE(4),
       1 PRD(95),YML(95),A1(5),SIGMA1(5),YFIT(95),
       2 SIGMAB(10),SIGMAG(10),
       3 SIGYML(95),YT(10),YPD(10),SIGYM(95)
        REAL LFLO
        READ (5,9000)BOLA,TOLA,LFLO,RTLO,STEP,SIZE
        READ (5,9010)IDENT,MODE,MINT,MAXT,LIST2,LIST1,INT
        READ (5,9000)EMMIN,PROB
        IF (INT.EQ.2)  READ(5,9020) A1(1),A1(2),A1(3)
        PI=3.141592
        RAD=180./PI
        STEPY=1./STEP
        FLAMDA=0.001
        NTERMS=3
        AA1=A1(1)
        AA2=A1(2)
        AA3=A1(3)
        WRITE(7,9030)
        IA=(TOLA-BOLA)*STEPY+1.
        JA=(RTLO-LFLO)*STEPY+1.
        KA=MAXT-MINT+1
        WRITE(6,9040)
        WRITE(6,9050)
```

```
      DO 10 I=1,IA
      DO 10 J=1,JA
      DO 10 K=1,KA
      A(I,J,K)=0.
   10 CONTINUE
      NO=0
C
C
C     READ IN AN EARTHQUAKE FROM THE CATALOGUE
C
   20 READ(3,9060,END=140) IYEAR,TITLE,O3,EN,BO,IDEPTH,EM
      IF(IYEAR.LT.MINT.OR.IYEAR.GT.MAXT) GO TO 20
      IF(EM.LT.EMMIN) GO TO 20
      NO=NO+1
      DEPTH=IDEPTH
      ENL=TOLA
C
C     SORT EARTHQUAKE INTO APPROPRIATE CELL. CHECK IT DOES NOT LIE OUTSIDE
C     RADIAL DISTANCE SIZE FROM CELL GRID POINT. STORE EARTHQUAKE
C     MAGNITUDE OR GROUND MOTION (COMPUTED AT CELL GRID POINT) PARAMETER
C     IN ARRAY A(I,J,K,)
C
      DO 130 I=1,IA
      E1=ENL-SIZE
      E2=ENL+SIZE
      IF(EN.GE.E2) GO TO 120
      IF(EN.LT.E1) GO TO 120
      BOY=LFLO
      DO 110 J=1,JA
      COR=COS(ENL*(1./RAD))
C
C     CORRECT CELL WIDTH SIZE FOR LATITUDE
      B1=BOY-(SIZE/COR)
      B2=BOY+(SIZE/COR)
      IF(BO.LT.B1) GO TO 100
      IF(BO.GE.B2) GO TO 100
      CALL DIRCOS(EN,BO,AE,BE,CE)
      CALL DIRCOS(ENL,BOY,AP,BP,CP)
      S=(AE-AP)**2+(BE-BP)**2+(CE-CP)**2
      IF(S.GT.0.) GO TO 30
      DIST=1.0
      GO TO 40
   30 S=1.-S*0.5
      S=SQRT(1.-S*S)/S
      DIST=ATAN(S)*RAD*111.1
      RS=SIZE*111.1
C
C     CHECK EPICENTRAL DISTANCE FROM GRID POINT DOES NOT
C     EXCEED RADIAL DISTANCE SIZE, NOW IN KM
      IF(DIST.GT.RS) GO TO 100
   40 R=SQRT(DIST*DIST+DEPTH*DEPTH)
C
C     SELECT EARTHQUAKE PARAMETER TO BE USED
C
      GO TO (50,60,70,80),IDENT
C
C     THE FOLLOWING FORMULA USED FOR ACCELERATION
C     FROM (MAKROPOULOS,K.C.,1978)
C     A=2164*EXP(EM*0.7)*(R+20)**-1.80
   50 US=-1.80
      AMP=2164.*EXP(EM*0.7)*(R+20)**US
      GO TO 90
C
C     FOR VELOCITY THE FORMULA OF ORPHAL AND  LAHOUD (1974) IS USED
   60 US=-1.34
      AMP=0.726*10.**(0.52*EM)*(R**US)
      GO TO 90
C
C        ...AND FOR DISPLACEMENT
   70 US=-1.18
      AMP=0.0471*10.**(EM*0.57)*(R**US)
      GO TO 90
C
C        ...AND IF MAGNITUDE USED
   80 AMP=EM
   90 K=IYEAR-MINT+1
C
C     STORE THE CURRENT EXTREME VALUE FOR THIS YEAR AND CELL
      IF(AMP.GT.A(I,J,K)) A(I,J,K)=AMP
```

```
      IF(LIST1.EQ.1) WRITE(6,9070) K,IYEAR,TITLE,O3,EN,BO,IDEPTH,EM,
     1 DIST,R,AMP,A(I,J,K)
  100 BOY=BOY+STEP
  110 CONTINUE
  120 ENL=ENL-STEP
  130 CONTINUE
C
C     NOW GET ANOTHER EARTHQUAKE
C
      GO TO 20
  140 WRITE(6,9080) NO
C
C     ALL NO EARTHQUAKES OF THE CATALOGUE ARE NOW SORTED AND ANNUAL
C     EXTREMES FOR EACH CELL RETAINED. ARRAY A(I,J,K) NOW CONTAINS:
C     A(GRID LAT, GRID LONG, EARTHQUAKE PARAMETER ANNUAL EXTREME)
C
C
      GO TO (150,160,170,180),IDENT
  150 WRITE(6,9090)
      GO TO 190
  160 WRITE(6,9100)
      GO TO 190
  170 WRITE(6,9110)
      GO TO 190
  180 WRITE(6,9120)
  190 CONTINUE
      WRITE(6,9130) RS
      WRITE(6,9230)
C
C     SET PROBABILITY LEVELS ON GUMBEL I HAZARD ASSESSMENTS
C
      IF(INT.EQ.2) GO TO 210
      RYEAR=25
      PROB1M=1-PROB
      PROB=ALOG(-ALOG(PROB1M))
      PROB1M=100.*PROB1M
      DO 200 I=1,4
      RY(I)=ALOG(RYEAR)
      RYEAR=RYEAR*2.
  200 CONTINUE
  210 ENL=TOLA
C
C
C     START PROCESSING CELLS
C
      DO 370 I=1,IA
      BOY=LFLO
      DO 360 J=1,JA
      WRITE(6,9140) ENL,BOY
      L=0
C
C     L COUNTS THE NUMBER OF OBSERVED ANNUAL EXTREMES IN A CELL (10 IS MIN)
      DO 230 K=1,KA
      IF(A(I,J,K).EQ.0.) GO TO 220
      L=L+1
  220 YM(K)=A(I,J,K)
  230 CONTINUE
      IF(L.LT.10) GO TO 350
      SK=L+1
      LL=SK-1
      IF(LIST2.EQ.1) WRITE(6,9150)
C
C     RANK THE EXTREME VALUES IN THE CELL
      CALL RANK(KA,YM,Y)
C
C     M IS THE NUMBER OF MISSING EXTREMES IN THE CELL
      M=KA-L
      L=0
C
C     EVALUATE THE OBSERVED PROBABILITY FOR EACH EXTREME
C     USING GRINGORTEN'S (1963) FORMULA
      DO 240 K=1,KA
      LYEAR=MINT+K-1
      IF(A(I,J,K).EQ.0.) GO TO 240
      L=L+1
      SJ=(MAXT-MINT+1)-SK+1+L-0.44
C
C     EVALUATE OBSERVED PROBABILITY PRO1 FOR AN INDIVIDUAL EXTREME
      PRO1=SJ/(MAXT-MINT+1.12)
```

```
      GY(L)=-ALOG(-ALOG(PRO1))
      M=M+1
      PRO(L)=PRO1
      YML1=Y(M)
      YM(L)=YML1
      YML(L)=YML1
      GYK=GY(L)
      IF(LIST2.EQ.1) WRITE(6,9160) K,LYEAR,A(I,J,K),YML1,GYK,PRO1,L
  240 CONTINUE
C
C     FOR ONE CELL THE ARRAY PRO(L) NOW CONTAINS THE OBSERVED PROBABILITY
C     OF BEING AN EXTREME, GY(L) CONTAINS THE REDUCED VARIATE AND YM(L)
C     CONTAINS THE RANKED EXTREMES
C
      A1(1)=YML1+.3
      WRITE(6,9170) L,MINT,LYEAR
C
C     ASSIGN SPECIFIC WEIGHTS TO THE DATA IF REQUIRED
      IF(MODE.NE.1) GO TO 290
      GO TO (250,290,290,270),IDENT
  250 LL=SK-1
      DO 260 II=1,LL
C
C     FOR PEAK ACCELERATION AFTER MCGUIRE (1974)
      SIGYM(II)=0.6672*YM(II)
  260 CONTINUE
      GO TO 300
  270 LL=SK-1
C
C     WEIGHTS FOR MAGNITUDES ACCORDING TO THEIR SIZE
      DO 280 II=1,LL
      IF(YML(II).LE.4.0) SIGYML(II)=0.4
      IF(YML(II).GT.4.0.AND.YML(II).LE.5.0) SIGYML(II)=0.3
      IF(YML(II).GT.5.0.AND.YML(II).LE.6.0) SIGYML(II)=0.2
      IF(YML(II).GT.6.0) SIGYML(II)=0.1
  280 CONTINUE
C
C     CHOOSE GUMBEL I OR GUMBEL III ANALYSIS
C
  290 GO TO (300,320),INT
C
C     FIT GUMBEL I TO MAGNITUDE EXTREME VALUES
  300 IF(IDENT.EQ.4)  CALL LINFIT(GY,YML,SIGYML,L,MODE,ALFA,SIGA,BHTA,
     1SIGB,R)
C
C     FIT GUMBEL I TO GROUND MOTION EXTREME VALUES
      IF(IDENT.NE.4)  CALL LINFIT(GY,YM,SIGYM,L,MODE,ALFA,SIGA,BHTA,
     1SIGB,R)
      YMOD=ALFA
      YP=ALFA-(PROB*BHTA)
      WRITE(6,9040)
      KYEAR=1
      WRITE(6,9180) YMOD,YP,KYEAR
      KYEAR=25
      DO 310 N=1,4
      RB=RY(N)*BHTA
      YT(N)=YMOD+RB
      YPD(N)=YP+RB
      WRITE(6,9190) YT(N),YPD(N),KYEAR
      KYEAR=KYEAR*2
  310 CONTINUE
C
C     OUTPUT GUMBEL I HAZARD RESULTS FOR CELL
      WRITE(6,9200)
      WRITE(6,9210) ENL,BOY,YMOD,(YT(N),N=1,4)
      WRITE(6,9220) ENL,BOY,YP,(YPD(N),N=1,4),PROB1M
      WRITE(6,9040)
      WRITE(6,9230)
C
C     FIT GUMBEL III TO MAGNITUDE EXTREME VALUES
      GO TO 350
  320 CALL CURFIT(PRO,YML,SIGYML,LL,NTERMS,MODE,A1,SIGMA1,
     1FLAMDA,YFIT,CHISQR,SIGMAB,SIGMAG)
  330 Z=CHISQR
      CALL CURFIT(PRO,YML,SIGYML,LL,NTERMS,MODE,A1,SIGMA1,
     1FLAMDA,YFIT,CHISQR,SIGMAB,SIGMAG)
      U=Z-CHISQR
      IF(U.GT.0.001) GO TO 330
C
```

```
C     OUTPUT GUMBEL III PARAMETERS AND HAZARD RESULTS FOR CELL
      WRITE(6,9240) CHISQR
      WRITE(6,9250) A1(1),SIGMA1(1)
      WRITE(6,9260) A1(2),SIGMA1(2)
      WRITE(6,9270) A1(3),SIGMA1(3)
      DO 340 JJ=1,3
      WRITE(6,9280) JJ,SIGMAB(JJ),JJ,SIGMAG(JJ)
  340 CONTINUE
C
C     ANNUAL MODE
      ZM1=A1(1)-(A1(1)-A1(2))*(1.-A1(3))**A1(3)
C
C     N-YEARS MODE  (75YEARS)
      ZMN=A1(1)-(A1(1)-A1(2))*((1.-A1(3))/75.)**A1(3)
C
C     MAGN.WITH 90% PROB. TO BE THE MAX.ANNUAL MAGN.
      ZM190=A1(1)-(A1(1)-A1(2))*((-ALOG(.90))**A1(3))
C
C     MAGN WITH 90% PROB.TO BE THE MAX. IN THE NEXT N YEARS (75YRS)
      ZMN90=A1(1)-(A1(1)-ZM190)/(75.**A1(3))
C
      WRITE(6,9290) ZM1,ZMN
      WRITE(7,9300) ENL,BOY,ZM1,ZM190,ZMN,ZMN90,YML1
      A1(1)=AA1
      A1(2)=AA2
      A1(3)=AA3
  350 BOY=BOY+STEP
  360 CONTINUE
      ENL=ENL-STEP
  370 CONTINUE
C
C
 9000 FORMAT(6F6.2)
 9010 FORMAT(7I5)
 9020 FORMAT(3F7.3)
 9030 FORMAT(3X,'GEOGR.COORD.    AN.MODE  90%NBE 75YMODE',
     1' 75Y90%NBE MAXOBS')
 9040 FORMAT(1H )
 9050 FORMAT(/5X,'HAZARD ANALYSIS IN A GIVEN REGION BASED ON THE'/,
     15X,'GUMBEL S STATISTICAL THEORY OF EXTREME VALUES, '/)
 9060 FORMAT(1X,I4,4A4,F4.1,2F8.2,I5,9X,F3.1)
 9070 FORMAT(1X,I2,I4,4A4,F4.1,2F8.2,I5,F5.1,2F7.1,2F7.2)
 9080 FORMAT(5X,'NUMBER OF PROCESSED EARTHQUAKE DATA',I5)
 9090 FORMAT(5X,'HAZARD ANALYSIS BASED ON THE ACCELERATION VALUES
     1   IN CM/SEC**2')
 9100 FORMAT(5X,'HAZARD ANALYSIS BASED ON THE VELOCITY VALUES IN'
     1' CM/SEC')
 9110 FORMAT(5X,'HAZARD ANALYSIS BASED ON THE DISPLACEMENT VALUES',
     1' IN CM')
 9120 FORMAT(5X,'HAZARD ANALYSIS BASED ON MAGNITUDE VALUES')
 9130 FORMAT(5X,'SIZE OF EARTHQUAKE SOURCE REGION',
     1F8.2,'KMS',/)
 9140 FORMAT(/3X,F6.2,2H N,2X,F6.2,2H E,/4X,17(1H*))
 9150 FORMAT(13X,'K YEAR',5X,'AMP.',5X,'RANKED',6X,'G(Y)',
     15X,'PROB.',3X,'L')
 9160 FORMAT(10X,I4,I5,4F10.3,I4)
 9170 FORMAT(/19X,'NUMBER OF OBSERVED SHOCKS',I5,/,19X,
     1'BETWEEN',I7,'  -  ',I5,2X,'YEARS')
 9180 FORMAT(/16X,2(2X,F9.2),I5,'  YEAR')
 9190 FORMAT( 16X,2(2X,F9.2),I5,'  YEARS')
 9200 FORMAT(3X,'LAT',4X,'LON',5X,'1YEAR',3X,'25YRS',3X,'50YRS',2X,'100
     1YRS',2X,'200YRS')
 9210 FORMAT(7(1X,F7.2),5X,'MODE')
 9220 FORMAT(7(1X,F7.2),2X,F3.0,'% PR. OF NBE')
 9230 FORMAT(1H1)
 9240 FORMAT(1H ,'FINAL CHISQR=',F10.5)
 9250 FORMAT(1H ,'W=',F7.4,3X,'SD.OF W=',F7.4)
 9260 FORMAT(1H ,'U=',F7.4,3X,'SD.OF U=',F7.4)
 9270 FORMAT(1H ,'L=',F7.4,3X,'SD.OF L=',F7.4)
 9280 FORMAT(1H ,'COV2',I1,'=',F7.4,2X,'COV1',I1,'=',F7.4)
 9290 FORMAT(1H ,'ANNUAL MODE=',F6.2,2X,'75 YEAR MODE=',F6.2)
 9300 FORMAT(7F8.2)
      STOP
      END
C
C
      SUBROUTINE DIRCOS(RLA,RLO,A,B,C)
C
C     EVALUATE DIRECTION COSINES FOR DISTANCE CALCULATIONS
```

```
C
      PI=3.141592
      RAD=PI/180.
      EN=RLA*RAD
      BO=RLO*RAD
      EN=ATAN(0.99238*SIN(EN)/COS(EN))
      C=SIN(EN)
      X=-COS(EN)
      D=SIN(BO)
      E=-COS(BO)
      A=X*E
      B=-D*X
      RETURN
      END
C
C
      SUBROUTINE RANK(N,Y,X)
C
C     RANK EXTREME VALUES IN ASCENDING SIZE
C
C     Y IS THE OBSERVED EXTREME VALUES
C     X IS THE EXTREME VALUES AFTER RANKING
C
      DIMENSION X(95),Y(95)
      YMAX=1.E38
      X1=YMAX
      DO 20 J=1,N
      YMIN=1.E37
      DO 10 I=1,N
      IF(Y(I).GE.YMIN) GO TO 10
      YMIN=Y(I)
      K=I
      IF(Y(I).GT.X1) GO TO 10
      YMIN=Y(I)
   10 CONTINUE
      X(J)=YMIN
      X1=YMIN
      Y(K)=YMAX
   20 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE LINFIT(X,Y,SIGMAY,NPTS,MODE,A,SIGA,B,SIGB,R)
C
C     PERFORM LEAST-SQUARES FIT TO DATA WITH STRAIGHT LINE GUMBEL I
C     EXTREME VALUE DISTRIBUTION
C
C     LEAST-SQUARES STRAIGHT LINE FITTING PROGRAM DERIVED FROM
C     BEVINGTON(1969), DATA REDUCTION AND ERROR ANALYSIS FOR THE
C     PHYSICAL SCIENCES, MCGRAW-HILL INC.
C
      DOUBLE PRECISION SUM,SUMX,SUMY,SUMX2,SUMXY,SUMY2
      DOUBLE PRECISION XI,YI,WEIGHT,DELTA,VARNCE
      DIMENSION X(95),Y(95),SIGMAY(95)
      SUM=0.0
      SUMX=0.0
      SUMY=0.0
      SUMX2=0.0
      SUMXY=0.0
      SUMY2=0.0
      DO 70 I=1,NPTS
      XI=X(I)
      YI=Y(I)
      IF(MODE) 10,40,50
   10 IF(YI) 30,40,20
   20 WEIGHT=1./YI
      GO TO 60
   30 WEIGHT=1./(-YI)
      GO TO 60
   40 WEIGHT=1.
      GO TO 60
   50 WEIGHT=1./SIGMAY(I)**2
   60 SUM=SUM+WEIGHT
      SUMX=SUMX+WEIGHT*XI
      SUMY=SUMY+WEIGHT*YI
      SUMX2=SUMX2+WEIGHT*XI*XI
      SUMXY=SUMXY+WEIGHT*XI*YI
      SUMY2=SUMY2+WEIGHT*YI*YI
```

```
   70 CONTINUE
      DELTA=SUM*SUMX2-SUMX*SUMX
      A=(SUMX2*SUMY-SUMX*SUMXY)/DELTA
      B=(SUMXY*SUM-SUMX*SUMY)/DELTA
      IF(MODE) 80,90,80
   80 VARNCE=1.
      GO TO 100
   90 C=NPTS-2
      VARNCE=(SUMY2+A*A*SUM+B*B*SUMX2-2.*(A*SUMY+B*SUMXY-A*B*SUMX))/C
  100 SIGA=DSQRT(VARNCE*SUMX2/DELTA)
      SIGB=DSQRT(VARNCE*SUM/DELTA)
      R=(SUM*SUMXY-SUMX*SUMY)/DSQRT(DELTA*(SUM*SUMY2-SUMY*SUMY))
      WRITE(6,200) A,SIGA,B,SIGB,R
  200 FORMAT(1H ,'U=',F7.4,2X,'S.D.OF U=',F6.4,2X,'1/A=',F7.4,
     1 2X,'S.D.OF 1/A=',F6.4,2X,'R=',F6.4)
      RETURN
      END
C
C
      SUBROUTINE CURFIT(X,Y,SIGMAY,NPTS,NTERMS,MODE,A,
     1SIGMAA,FLAMDA,YFIT,CHISQR,SIGMAB,SIGMAG)
C
C     PERFORM LEAST SQUARES FIT TO DATA WITH NON-LINEAR GUMBEL III
C     EXTREME VALUE DISTRIBUTION
C
C     NON-LINEAR LEAST-SQUARES FITTING PROGRAM DERIVED FROM
C     BEVINGTON(1969), DATA REDUCTION AND ERROR ANALYSIS FOR THE
C     PHYSICAL SCIENCES, MCGRAW-HILL INC.
C
C     NEEDS SUBROUTINES:FDERIV,MATINV
C     NEEDS FUNCTIONS:FUNCTN,FCHISQ
C
      DOUBLE PRECISION ARRAY
      DIMENSION X(95),Y(95),SIGMAY(95),A(10),SIGMAA(10),
     1YFIT(95),SIGMAB(10),SIGMAG(10)
      DIMENSION WEIGHT(95),ALPHA(10,10),BETA(10),DERIV(10),
     1ARRAY(10,10),B(10)
      NFREE=NPTS-NTERMS
      IF(NFREE) 10,10,20
   10 CHISQR=0.0
      GO TO 230
   20 DO 80 II=1,NPTS
      IF(MODE) 30,60,70
   30 IF(Y(II)) 50,60,40
   40 WEIGHT(II)=1./Y(II)
      GO TO 80
   50 WEIGHT(II)=1./(-Y(II))
      GO TO 80
   60 WEIGHT(II)=1.
      GO TO 80
   70 WEIGHT(II)=1./SIGMAY(II)**2
   80 CONTINUE
      DO 90 J=1,NTERMS
      BETA(J)=0.0
      DO 90 K=1,J
      ALPHA(J,K)=0.0
   90 CONTINUE
      DO 110 IK=1,NPTS
      CALL FDERIV(X,IK,A,NTERMS,DERIV)
      DO 100 J=1,NTERMS
      BETA(J)=BETA(J)+WEIGHT(IK)*(Y(IK)-FUNCTN(X,IK,A))*DERIV(J)
      DO 100 K=1,J
      ALPHA(J,K)=ALPHA(J,K)+WEIGHT(IK)*DERIV(J)*DERIV(K)
  100 CONTINUE
  110 CONTINUE
      DO 120 J=1,NTERMS
      DO 120 K=1,J
      ALPHA(K,J)=ALPHA(J,K)
  120 CONTINUE
  130 DO 140 I=1,NPTS
      YFIT(I)=FUNCTN(X,I,A)
  140 CONTINUE
      CHISQ1=FCHISQ(Y,SIGMAY,NPTS,NFREE,MODE,YFIT)
  150 DO 170 J=1,NTERMS
      DO 160 K=1,NTERMS
      ARRAY(J,K)=ALPHA(J,K)/SQRT(ALPHA(J,J)*ALPHA(K,K))
  160 CONTINUE
      ARRAY(J,J)=1.+FLAMDA
  170 CONTINUE
```

```
      CALL MATINV(ARRAY,NTERMS,DET)
      DO 180 J=1,NTERMS
      B(J)=A(J)
      DO 180 K=1,NTERMS
      B(J)=B(J)+BETA(K)*ARRAY(J,K)/SQRT(ALPHA(J,J)*ALPHA(K,K))
  180 CONTINUE
      DO 190 I=1,NPTS
      YFIT(I)=FUNCTN(X,I,B)
  190 CONTINUE
      CHISQR=FCHISQ(Y,SIGMAY,NPTS,NFREE,MODE,YFIT)
      IF(CHISQ1-CHISQR) 200,210,210
  200 FLAMDA=10.*FLAMDA
      GO TO 150
  210 DO 220 J=1,NTERMS
      A(J)=B(J)
      SIGMAB(J)=ARRAY(J,2)/SQRT(ALPHA(J,J)*ALPHA(2,2))
      SIGMAG(J)=ARRAY(J,1)/SQRT(ALPHA(J,J)*ALPHA(1,1))
      SIGMAA(J)=DSQRT(ARRAY(J,J)/ALPHA(J,J))
  220 CONTINUE
      FLAMDA=FLAMDA/10.
  230 RETURN
      END
C
C
      SUBROUTINE FDERIV(X,I,A,NTERMS,DERIV)
C
C     EVALUATE PARTIAL DERIVATIVES OF GUMBEL III FUNCTION WITH RESPECT
C     TO ITS THREE PARAMETERS PRIOR TO LEAST SQUARES FITTING TO DATA
C     WITH GUMBEL III
C
      DIMENSION X(95),A(10),DERIV(10)
      XI=X(I)
      Z1=-ALOG(XI)
      Z2=Z1**A(3)
      DERIV(1)=1.-Z2
      DERIV(2)=Z2
      DERIV(3)=-(A(1)-A(2))*Z2*ALOG(Z1)
      RETURN
      END
C
C
      SUBROUTINE MATINV(ARRAY,NORDER,DET)
C
C     INVERT A SYMMETRICAL MATRIX AND CALCULATE ITS DETERMINANT
C
      DOUBLE PRECISION ARRAY,AMAX,SAVE
      DIMENSION ARRAY(10,10),IK(10),JK(10)
      DET=1.
      DO 190 K=1,NORDER
      AMAX=0.0
   10 DO 30 I=K,NORDER
      DO 30 J=K,NORDER
      IF(DABS(AMAX)-DABS(ARRAY(I,J))) 20,20,30
   20 AMAX=ARRAY(I,J)
      IK(K)=I
      JK(K)=J
   30 CONTINUE
      IF(AMAX) 50,40,50
   40 DET=0.0
      GO TO 260
   50 I=IK(K)
      IF(I-K) 10,80,60
   60 DO 70 J=1,NORDER
      SAVE=ARRAY(K,J)
      ARRAY(K,J)=ARRAY(I,J)
      ARRAY(I,J)=-SAVE
   70 CONTINUE
   80 J=JK(K)
      IF(J-K) 10,110,90
   90 DO 100 I=1,NORDER
      SAVE=ARRAY(I,K)
      ARRAY(I,K)=ARRAY(I,J)
      ARRAY(I,J)=-SAVE
  100 CONTINUE
  110 DO 130 I=1,NORDER
      IF(I-K) 120,130,120
  120 ARRAY(I,K)=-ARRAY(I,K)/AMAX
  130 CONTINUE
      DO 160 I=1,NORDER
```

```
      DO 160 J=1,NORDER
      IF(I-K) 140,160,140
  140 IF(J-K) 150,160,150
  150 ARRAY(I,J)=ARRAY(I,J)+ARRAY(I,K)*ARRAY(K,J)
  160 CONTINUE
      DO 180 J=1,NORDER
      IF(J-K) 170,180,170
  170 ARRAY(K,J)=ARRAY(K,J)/AMAX
  180 CONTINUE
      ARRAY(K,K)=1./AMAX
      DET=DET*AMAX
  190 CONTINUE
      DO 250 L=1,NORDER
      K=NORDER-L+1
      J=IK(K)
      IF(J-K) 220,220,200
  200 DO 210 I=1,NORDER
      SAVE=ARRAY(I,K)
      ARRAY(I,K)=-ARRAY(I,J)
      ARRAY(I,J)=SAVE
  210 CONTINUE
  220 I=JK(K)
      IF(I-K) 250,250,230
  230 DO 240 J=1,NORDER
      SAVE=ARRAY(K,J)
      ARRAY(K,J)=-ARRAY(I,J)
      ARRAY(I,J)=SAVE
  240 CONTINUE
  250 CONTINUE
  260 RETURN
      END
C
C
      FUNCTION FUNCTN(X,IM,A)
C
C     EVALUATE THE GUMBEL III FUNCTION AT THE IM'TH TERM
C
C     X IS THE INDEPENDENT VARIABLE TAKEN AS THE EXTREME VALUE
C       DATUM 'PLOTTING POINT' PROBABILITY
C     A CONTAINS THE THREE GUMBEL III PARAMETERS
C
      DIMENSION X(95),A(10)
      XI=X(IM)
      Z1=-ALOG(XI)
      Z2=Z1**A(3)
      Z3=(A(1)-A(2))*Z2
      FUNCTN=A(1)-Z3
      RETURN
      END
C
C
      FUNCTION FCHISQ(Y,SIGMAY,NPTS,NFREE,MODE,YFIT)
C
C     EVALUATE REDUCED CHI SQUARE FOR FIT OF GUMBEL III TO DATA
C
      DOUBLE PRECISION CHISQ,WEIGHT
      DIMENSION Y(95),SIGMAY(95),YFIT(95)
      CHISQ=0.0
      IF(NFREE) 10,10,20
   10 FCHISQ=0.0
      GO TO 90
   20 DO 80 I=1,NPTS
      IF(MODE) 30,60,70
   30 IF(Y(I)) 50,60,40
   40 WEIGHT=1./Y(I)
      GO TO 80
   50 WEIGHT=1./(-Y(I))
      GO TO 80
   60 WEIGHT=1.
      GO TO 80
   70 WEIGHT=1./SIGMAY(I)**2
      CHISQ=CHISQ+WEIGHT*(Y(I)-YFIT(I))**2
   80 CONTINUE
      FREE=NFREE
      FCHISQ=CHISQ/FREE
   90 RETURN
      END
```

APPENDIX 2

Listing of specimen earthquake catalog data; stream 3 input to HAZAN

These earthquakes have been selected from a larger catalog as test input data and are selected only to illustrate an application of HAZAN without extensive earthquake catalog input.

1901 OCT 25	16 18 30.0	37.00	22.20	20	5.4
1901 DEC 24	23 18 0.0	37.20	22.20	15	5.8
1902 APR 11	18 35 0.0	38.50	23.50	24	5.8
1902 AUG 02	05 38 0.0	38.50	21.80	20	5.6
1903 JUL 21	13 03 0.0	38.20	21.80	20	5.6
1904 APR 05	09 33 30.0	37.80	22.20	5	5.5
1909 MAY 30	06 14 0.0	38.25	22.20	20	6.0
1909 JUN 13	09 15 30.0	38.30	22.00	24	5.5
1911 MAR 16	03 12 43.0	38.20	22.00	20	5.4
1911 SEP 20	23 24 28.0	37.50	22.50	20	5.2
1914 OCT 17	06 22 32.0	38.20	23.50	8	6.0
1916 MAY 20	22 14 0.0	38.20	23.20	28	5.5
1916 SEP 27	15 02 13.0	38.80	23.00	6	5.8
1917 DEC 24	09 13 58.2	38.65	21.86	15	5.8
1918 JAN 27	12 56 35.0	38.50	22.00	24	5.1
1919 OCT 25	17 54 0.5	38.28	23.72	44	5.0
1922 AUG 08	03 49 15.0	37.58	24.29	67	5.4
1922 NOV 11	22 13 10.5	37.84	22.03	32	5.2
1924 FEB 16	09 01 6.0	37.50	23.00	15	5.5
1925 APR 12	19 27 0.9	38.64	23.52	24	5.0
1925 JUL 06	12 15 54.3	37.79	21.94	70	5.8
1926 FEB 26	16 08 26.7	37.85	21.47	6	5.6
1928 APR 22	20 13 55.9	38.08	23.12	8	6.5
1930 APR 17	20 06 49.2	37.80	23.17	66	6.1
1931 JAN 04	00 00 52.5	38.22	23.27	8	5.7
1938 JUL 20	00 23 42.5	38.30	23.66	42	6.1
1938 SEP 18	03 50 40.9	38.27	22.47	53	5.9
1939 JUN 02	14 11 43.0	38.65	22.07	148	5.2
1944 JUL 30	04 00 45.6	37.14	22.27	85	5.6
1947 JUL 21	09 36 36.3	37.55	22.99	60	5.0
1948 SEP 11	08 52 44.0	37.38	23.28	88	6.2
1949 SEP 17	11 30 15.8	37.07	22.67	42	5.0
1949 OCT 04	17 33 33.9	38.63	22.08	111	5.0
1952 JUN 13	01 07 30.2	37.31	21.98	55	5.3
1953 SEP 05	14 18 46.0	37.88	23.17	18	5.7
1954 APR 17	20 52 51.5	37.99	22.98	19	5.1
1955 APR 13	20 45 51.3	37.29	22.50	19	5.2
1957 MAY 29	18 39 27.2	37.62	23.42	120	5.3
1958 NOV 15	05 42 40.5	37.45	21.73	31	5.5
1959 MAR 29	23 07 24.5	37.39	23.81	61	4.6
1959 AUG 16	18 42 9.5	37.23	22.38	63	5.1
1962 JAN 19	19 38 2.7	38.35	22.25	35	5.3
1962 AUG 28	10 59 57.4	37.80	22.88	95	6.6
1962 OCT 04	19 46 12.1	37.93	22.36	53	5.0
1964 JUL 17	02 34 26.7	38.05	23.63	155	6.0
1964 DEC 01	10 21 3.3	38.53	22.45	48	4.7
1965 MAR 31	09 47 26.3	38.38	22.26	45	6.6
1965 APR 05	03 12 54.6	37.75	22.00	34	6.0
1965 JUL 06	03 18 42.1	38.37	22.40	18	6.4
1966 JAN 02	23 12 18.0	37.67	23.18	12	4.7
1966 FEB 17	10 41 25.8	38.89	21.88	38	5.3
1966 SEP 01	14 22 56.9	37.46	22.16	15	5.4
1967 JAN 04	05 58 52.5	38.37	22.04	1	5.5
1967 JUN 12	02 51 5.8	38.15	22.77	35	5.0
1968 JUL 04	21 47 53.6	37.76	23.23	20	5.5
1969 JAN 13	05 46 40.4	38.31	22.52	46	4.9
1969 OCT 02	23 13 40.6	38.47	22.29	45	4.7
1970 APR 08	13 50 28.3	38.34	22.56	23	6.2
1970 APR 20	15 39 31.6	38.27	22.66	38	5.3
1970 OCT 01	22 38 37.2	38.02	22.77	43	5.3
1971 FEB 09	21 20 35.3	38.13	22.77	40	4.4
1971 MAR 15	15 23 19.8	37.29	24.14	41	4.7
1971 MAY 26	07 09 26.0	37.10	21.70	33	4.9
1971 SEP 11	02 03 11.5	38.87	22.31	5	4.4
1971 SEP 29	21 02 34.3	37.02	23.28	60	4.4
1972 APR 26	21 14 11.1	38.24	22.43	81	4.5
1972 JUN 15	00 33 24.9	38.34	22.20	33	5.1
1972 SEP 13	04 13 19.7	37.96	22.38	75	6.2
1973 JAN 10	03 24 12.0	37.69	21.42	45	4.9
1973 MAR 21	11 25 52.1	37.47	23.67	43	4.2
1974 NOV 14	13 22 34.7	38.50	23.08	27	5.0
1974 NOV 14	14 26 46.6	38.48	23.01	6	5.1
1974 NOV 14	15 29 46.8	38.50	23.15	35	5.0

1975	JAN	08	19	32	34.1	38.24	22.65	26		5.7
1975	APR	04	05	16	16.5	38.11	21.98	56		5.7
1975	OCT	12	08	23	12.6	37.91	23.12	35		5.0
1976	JAN	01	00	04	6.0	38.42	21.72	18		4.7
1976	JAN	14	10	31	2.3	38.39	21.95	10		4.6
1976	JUN	20	04	51	17.0	38.53	22.12	51		4.7
1976	DEC	30	15	12	38.4	37.83	22.85	35		4.9
1977	JAN	16	09	16	48.8	37.84	22.95	45		4.9
1977	DEC	29	16	52	58.8	38.29	22.25	37		5.0
1978	APR	05	04	50	45.0	37.68	23.15	31		4.8
1978	APR	08	06	22	27.1	36.95	23.24	48		4.9
1979	MAR	13	13	48	58.7	38.54	24.29	19		4.7
1979	JUN	26	15	34	30.6	38.81	23.27	4		4.6
1979	JUL	02	15	43	22.5	38.08	22.90	44		4.6
1979	DEC	01	13	34	30.1	37.26	21.73	43		5.0
1980	FEB	28	23	45	16.6	38.17	23.23	30		4.8
1980	JUL	02	17	10	38.3	38.14	22.00	20		5.0
1981	FEB	24	20	53	37.0	38.23	22.97	18		6.6
1981	FEB	25	02	35	53.5	38.17	23.12	30		6.3
1981	MAR	04	21	58	7.2	38.24	23.26	21		6.4

APPENDIX 3

Example of selected program output

This output uses the earthquake catalog test data of Appendix 2 as input stream 3 and input stream 5 being four cards containing the specimen numerical data given as examples in main text.

(a) Output stream 6: table of annual extreme magnitudes and ranked extremes, third type asymptote distribution parameters and their uncertainties and covariances.

```
HAZARD ANALYSIS IN A GIVEN REGION BASED ON THE
GUMBEL S STATISTICAL THEORY OF EXTREME VALUES,

NUMBER OF PROCESSED EARTHQUAKE DATA   93
HAZARD ANALYSIS BASED ON MAGNITUDE VALUES
SIZE OF EARTHQUAKE SOURCE REGION  111.10KMS


37.95 N   22.92 E
*****************
```

K	YEAR	AMP.	RANKED	G(Y)	PROB.	L
2	1901	5.800	4.200	0.212	0.445	1
3	1902	5.800	4.400	0.246	0.457	2
5	1904	5.500	4.600	0.280	0.470	3
10	1909	6.000	4.800	0.314	0.482	4
12	1911	5.400	4.900	0.349	0.494	5
15	1914	6.000	4.900	0.384	0.506	6
17	1916	5.800	5.000	0.420	0.518	7
19	1918	5.100	5.000	0.456	0.530	8
20	1919	5.000	5.000	0.492	0.543	9
23	1922	5.200	5.000	0.529	0.555	10
25	1924	5.500	5.000	0.567	0.567	11
26	1925	5.800	5.100	0.605	0.579	12
29	1928	6.500	5.100	0.644	0.591	13
31	1930	6.100	5.100	0.683	0.604	14
32	1931	5.700	5.100	0.724	0.616	15
39	1938	6.100	5.200	0.765	0.628	16
40	1939	5.200	5.200	0.807	0.640	17
45	1944	5.600	5.200	0.850	0.652	18
48	1947	5.000	5.300	0.894	0.664	19
49	1948	6.200	5.300	0.940	0.677	20
50	1949	5.000	5.400	0.986	0.689	21
53	1952	5.300	5.400	1.035	0.701	22
54	1953	5.700	5.500	1.084	0.713	23
55	1954	5.100	5.500	1.136	0.725	24
56	1955	5.200	5.500	1.189	0.737	25
58	1957	5.300	5.500	1.244	0.750	26
60	1959	5.100	5.600	1.302	0.762	27
63	1962	6.600	5.700	1.362	0.774	28
65	1964	6.000	5.700	1.425	0.786	29
66	1965	6.600	5.700	1.491	0.798	30
67	1966	5.400	5.800	1.560	0.811	31
68	1967	5.500	5.800	1.634	0.823	32
69	1968	5.500	5.800	1.712	0.835	33
70	1969	4.900	5.800	1.796	0.847	34
71	1970	6.200	6.000	1.886	0.859	35
72	1971	4.400	6.000	1.983	0.871	36
73	1972	6.200	6.000	2.089	0.884	37

74	1973	4.200	6.100	2.207	0.896	38
75	1974	5,100	6.100	2.337	0.908	39
76	1975	5.700	6.200	2.486	0.920	40
77	1976	4.900	6.200	2.658	0.932	41
78	1977	5.000	6.200	2.862	0.944	42
79	1978	4.800	6.500	3.116	0.957	43
80	1979	4.600	6.600	3.452	0.969	44
81	1980	5.000	6.600	3.954	0.981	45
82	1981	6.600	6.600	4.985	0.993	46

```
            NUMBER OF OBSERVED SHOCKS   46
            BETWEEN   1900  -   1981  YEARS
FINAL CHISQR=    0.24152
W= 6.8473    SD.OF W= 0.1374
U= 4.3051    SD.OF U= 0.1280
L= 0.5492    SD.OF L= 0.0854
COV21= 0.0111   COV11= 0.0189
COV22= 0.0164   COV12= 0.0111
COV23=-0.0089   COV13=-0.0110
ANNUAL MODE=  5.21  75 YEAR MODE=  6.69
```

(b) Output stream 7: various hazard values.

GEOGR.COORD.		AN.MODE	90%NBE	75YMODE	75Y90%NBE	MAXOBS
37.95	22.42	5.08	6.07	6.71	6.86	6.60
37.95	22.92	5.21	6.11	6.69	6.78	6.60
37.95	23.42	4.96	6.00	6.72	6.79	6.60