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Seismic Risk of Circum-Pacific Earthquakes I. Strain Energy Release

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Abstract - Commonly used earthquake "whole process" frequency-magnitude and strain energy-magnitude laws are merged to obtain an analytic expression for an upper bound magnitude to regional earthquake occurrence M_3, which is expressed primarily in terms of the annual maximum magnitude M_1 and the magnitude equivalent of the annual average total strain energy release M_2. Values of M_3 are also estimated graphically from cumulative strain energy release diagrams. Both methods are illustrated by application to the high seismicity of the circum-Pacific belt, using Duda's (1965) data and regionalisation. Values of M_3 obtained analytically, with their uncertainties, are in agreement with those obtained graphically. Empirical relations are then obtained between M_1, M_2, and M_3, which could be of general assistance in regional seismic risk considerations if they are found to be of a universal nature. For instance, M_3 and M_2 differ by one magnitude unit in subregions of the circum-Pacific.

Key Words: Seismicity; Strain energy; Risk; Circum-Pacific.

1. Introduction

Reliable estimations of seismic risk and seismic hazard, and developments of means of mapping them, are among the research problems in seismology that urgently require answers.

Models of seismic risk usually consist of (a) empirical formulas based on available macroseismic data, (b) statistical distribution laws for earthquake occurrence in time and magnitude, and (c) attenuation laws describing the decay of seismic ground motion with focal distance.

The distribution of earthquake magnitudes in time and in size is generally investigated by (a) using the whole available data set of magnitudes (whole process) or (b) using only the extreme value magnitudes (part process). In this study the whole available data set is used for seismic risk evaluation; in

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our next paper the problem will be analysed using only extreme-magnitude earthquakes and GUMBEL's (1958) third-type asymptotic distribution.

When models using the whole process are applied to the experimental data, like the linear frequency–magnitude model of GUTENBERG and RICHTER (1944),

$$\log N(m) = a - bm, \tag{1}$$

where $N(m)$ is the mean number of earthquakes per unit volume and per unit time having magnitude greater than m, and a and b are zone-dependent constants, it becomes clear that they do not represent the real process for the large earthquakes. Many of the proposed alternative expressions, such as the quadratic frequency–magnitude formulas (e.g., MERZ and CORNELL, 1973; CORNELL and VANMARCKE, 1969), do not recognise the existence of an upper bound to the magnitude or do not identify what this upper bound might be. Several authors have considered the need for an upper bound to magnitude (for example, COSENTINO, FICARRA, and LUZIO, 1977), and a wide-ranging review of the literature is BÅTH's (1981). However, the introduction of an upper bound to magnitude without considering the dependency of such an upper bound on some physical parameters that characterize the region is not ideal and might contribute to errors in the evaluation of the other parameters of the distribution, and consequently to the eventual estimation of the seismic risk. An ad hoc introduction of an upper bound to magnitude occurrence might not help facilitate understanding of regional variations in earthquake occurrence and seismic risk.

In this study it will be demonstrated that using equation (1) and the energy–magnitude formula

$$\log E = A + Bm \tag{2}$$

it is possible to include an upper-bound truncation for magnitude of earthquakes and that this is necessary to maintain compatibility with finite strain energy release rates available to the earthquake process. It will be shown that this upper bound is then related to two easily obtained parameters—the earthquake magnitude that is the annual maximum, and the annual average total strain energy release. Furthermore, the value of such an upper bound will be found to illustrate the method for each of the circum-Pacific belt subregions of DUDA (1965); this will be done both analytically and graphically.

Formulas of the type of (2) have been used by themselves on many different occasions to calculate and map regional tectonic flux (for example, HÉDERVÁRI and PAPP, 1977, 1981), and its use here combines this physically interpretable approach with the statistical view of equation (1).

2. *Energy release and the upper bound for earthquake magnitude*

2.1. *Theoretical considerations*

Let

M_1 be the most probable annual maximum magnitude (mode), which from equation (1) for N/year = 1 is equal to a/b;

M_2 be the magnitude that corresponds to the mean annual rate of energy release; and

M_3 be the upper bound for the earthquake magnitude in the same region.

From relations (1) and (2) we have:

$$\ln N = a' - b'm \rightarrow N = e^{a'-b'm} \tag{3}$$

and

$$\ln E = A' + B'm \rightarrow E = e^{A'+B'm}, \tag{4}$$

where

$$a' = a \ln 10, \quad b' = b \ln 10, \quad A' = A \ln 10, \quad B' = B \ln 10. \tag{5}$$

The number of earthquakes per year with magnitude in the range dm is

$$dN(m) = b'e^{a'-b'm}\, dm. \tag{6}$$

The annual energy release for all earthquakes with magnitude in the range dm, $d\bar{E}$, is then

$$d\bar{E} = e^{A'+B'm} \cdot b'e^{a'-b'm}\, dm, \tag{7}$$

and the total annual energy release TE is

$$\begin{aligned} TE &= \int_{M_0}^{M_3} \bar{E}(m)\, dm = b' \int_{M_0}^{M_3} e^{A'+B'm} \cdot e^{a'-b'm}\, dm \\ &= b'e^{a'+A'} \cdot \frac{1}{B'-b'} \cdot e^{(B'-b')m} \Big|_{M_0}^{M_3}, \end{aligned} \tag{8}$$

where M_0 is the earthquake magnitude threshold. Equation (8) is equivalent to

$$TE = \frac{b'}{B'-b'} \cdot e^{a'+A'}[e^{(B'-b')M_3} - e^{(B'-b')M_0}]. \tag{9}$$

Usually $b \sim 1$ and $B = 1.44$ for surface wave magnitude M_s used in this study (BÅTH, 1958), so $B - b \sim 0.5$. Hence only the case when $B > b$, which is almost always that observed, will be considered hereafter. Using the definition of M_2 and noting that

$$M_1 = \frac{a'}{b'} = \frac{a}{b}, \tag{10}$$

then equation (9) becomes

$$e^{B'M_2} = \frac{b'}{B' - b'} \cdot e^{M_1 b'}[e^{(B'-b')M_3} - e^{(B'-b')M_0}]. \tag{11}$$

The magnitude M_2, equivalent to the mean annual rate of energy release, has now been introduced in (11), and it is worth noting that average properties of fluctuating processes are those known with greatest accuracy. From inspection of (11) the observed finite values of M_2 are compatible with noninfinite values of the upper bound M_3. In principle M_0 is limited to the left by the finite earthquake magnitude $M_0 = M_{\min}$, which is the smallest physically possible. With $B' - b' \sim 1.15$ the exponential term in M_3 will be several orders of magnitude larger than that in $M_{\min}$; in practise, with M_3 values around 8.5 the first term is four orders of magnitude larger than the second, even with an excessively high value of 5 for $M_{\min}$. Neglecting this term in M_0, equation (11) becomes

$$e^{B'M_2} = \frac{b'}{B' - b'} \cdot e^{M_1 b'} \cdot e^{(B'-b')M_3}. \tag{12}$$

From (12) we get

$$M_2 = \frac{b'}{B'} M_1 + \frac{B' - b'}{B'} + \frac{1}{B'} \ln \left(\frac{b'}{B' - b'} \right) \tag{13}$$

and

$$M_3 = \frac{1}{B' - b'} \left[B'M_2 - b'M_1 - \ln \left(\frac{b'}{B' - b'} \right) \right]. \tag{14}$$

Because the mean rate of energy release M_2 is finite (KNOPOFF and KAGAN, 1978) the upper bound magnitude M_3 truncating equation (1) also must now be finite in equation (13). This conclusion need not be generally true mathematically for other magnitude frequency distributions (e.g., Gaussian).

The relations (13) and (14), because of (5), become

$$M_2 = \frac{1}{B} \left[bM_1 + (B - b)M_3 + \log \left(\frac{b}{B - b} \right) \right] \tag{15}$$

and

$$M_3 = \frac{1}{B - b} \left[BM_2 - bM_1 - \log \left(\frac{b}{B - b} \right) \right]. \tag{16}$$

Alternatively, this may be written as

$$M_3 = C_1(b)M_2 + C_2(b)M_1 + C_3(b), \tag{17}$$

where

$$C_1(b) = \frac{B}{B-b}, \quad C_2(b) = \frac{-b}{B-b}, \quad C_3(b) = \frac{-1}{B-b}\log\left(\frac{b}{B-b}\right). \tag{18}$$

In addition to the previous relations it can be shown that

$$M_3 - M_2 = [C_1(b) - 1]M_2 + C_2(b)M_1 + C_3(b), \tag{19}$$

and because

$$C_1(b) - 1 = -C_2(b), \tag{20}$$

it follows that

$$M_3 - M_2 = -C_2(b)[M_2 - M_1] + C_3(b); \tag{21}$$

similarly,

$$M_3 - M_1 = C_1(b)[M_2 - M_1] + C_3(b). \tag{22}$$

Several significant features emerge from this analysis. The first feature to note is that a finite upper bound to the largest earthquake magnitude described by equation (1) is necessary to preserve a finite rate of energy release. [This latter assumption is considered eminently reasonable by most seismologists (BÅTH, 1981).] This is inescapable if the often assumed linear frequency–magnitude law applies for a given region. A second significant feature arises from equation (17). The upper bound M_3 is expressed here as a function of the most probable annual maximum magnitude M_1 and the mean annual rate of energy release M_2. M_3 further depends on the b value that characterizes the region seismotectonically. The relation between M_3 and b, coupled with the property of b being different from region to region (DUDA, 1965), leads to the conclusion that each region must also have its own upper bound for earthquake magnitudes expected to occur within that region.

Thus, for a given region where the a and b of equation (1) are known from the whole-process seismicity, it is a simple matter to calculate M_1 and M_2 from equations (10) and (2), respectively. Equation (17) can then be used to estimate the maximum magnitude earthquake M_3 that may occur within that region. From the above equations the uncertainties of M_1, M_2, and M_3 can also be estimated.

The stationarity of M_3 is of course related to the stationarity of the parameters a and b. Generally, the larger the number of earthquakes available for analysis, the more reliable are the estimates of a and b (DUDA, 1965), the average value of the strain energy release process M_2, and consequently the value of M_3.

2.2. *Graphical method of estimating M_2 and M_3*

The graphical method of estimating M_2 and M_3 for a given region centres on plotting the cumulative energy released as a function of time (for example, see GALANOPOULOS, 1972; MAKROPOULOS, 1978). It is based on the assumption that the rate of total energy accumulation and release in a given region remains fairly constant, provided that the period of observation is long enough to average out large short-term fluctuations. Both this approach and the application of equation (1) accept the common assumption of time invariance of the statistical seismic process (LOMNITZ, 1974). It is to be emphasized that stationarity of a process does not imply that equal temporal windows of the earthquake process are identical in their content; short-term averages will fluctuate, and indeed it is to be expected that pseudoclustering of events in space or time will occur if the strain energy release is time invariant and random (e.g., Poisson). It is therefore to be expected that use of the entire temporal extent of the catalogue will produce the best available estimate of the overall earthquake process. However, variations in seismicity have been mooted for different areas in the world—for example, in China (LEE, WU, and JACOBSON, 1976), and on the North Anatolian Fault in Turkey (AMBRASEYS, 1971). Available evidence is not conclusive and alternative views are available even for the commonly cited example of Turkey (DEWEY, 1976; SOYSAL, KOLCAK, and SIPAHIOGLU, 1982), the latter of whom conclude that seismicity in Turkey has not varied over the long term. It appears in general that the strain-producing forces do not change appreciably within a time span of a few hundred years, and it is reasonable to assume that in a given region the rate of strain or energy accumulation and the possibility of energy retention may be taken as a characteristic of the region. The translation from strain-producing force to strain energy release need not be uniformly constant. Hence the *total energy* that may be accumulated and released remains fairly constant, although the annual average of strain energy release may itself fluctuate about a mean value, depending on the temporal window considered.

Therefore, from the graph of cumulative energy released as a function of time (see Figure 1) we can derive:

1. The rate of energy released, calculated as the slope of the line SS'. This line connects the starting point S (zero energy) with the final point S' (E total). The slope ($\Delta E/\Delta T$) represents the annual rate of energy released, and the corresponding magnitude will be M_2.
2. Since the total energy that may be accumulated and released in a given region is taken as constant, the two lines BB' and CC' enveloping maximum and minimum epochs of energy released—that is, the lines that pass through the end point BB' and beginning point CC' of the active period PP'—should run parallel to each other and to SS'. Thus the vertical distance E_1E_2 between these two enveloping parallel lines

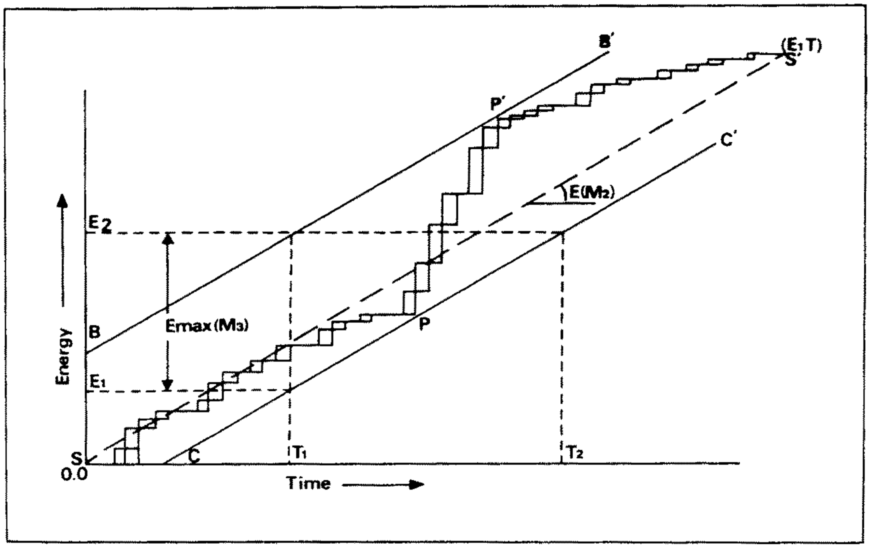

Figure 1. Explanatory diagram for graphical representation of the energy release method of calculating upper bound magnitude and mean annual energy release.

indicates the total amount of energy that may be released in the region. Hence, the vertical distance E_1E_2 (energy) is the upper limit E_{max} for the energy that can be observed in the region if the accumulated energy can be released by a single earthquake. The corresponding magnitude of such an earthquake must correspond to M_3 from this graphical model.

3. The horizontal distance T_1T_2 between the two parallel lines BB' and CC' indicates the minimum time Tr required for the accumulation of the maximum energy if there were no earthquakes in the meantime. This time interval will be called the *waiting time.*

3. *Application*

The circum-Pacific belt is chosen here as an area of application for both analytical and graphical methods. This region is seismically the most active in the world; only 24 percent of the earth's seismic energy release occurs outside of it. The data used are those compiled by DUDA (1965), whose catalogue is conveniently available. His catalogue supplies reliable and homogeneous data for large earthquakes ($M \geq 7.0$) over the whole world for the magnitudes and period considered. For our present purpose we make no attempt to correct the magnitudes available in the catalogue for any possible effects of saturation (KANAMORI, 1977), but point out that a systematic correction is neither generally available nor applicable, since except for great earthquakes (ABE and KANAMORI, 1980) the trend is not a consistent increase in magnitude to adjust for instrumentally saturated observations of long period surface waves. Following Duda's subdivision, the circum-Pacific belt is divided into eight subregions; the eighth is omitted here for reasons of insufficient data.

For each of the seven remaining subregions the constants a and b of the frequency-magnitude relation (1) and the annual energy release, using equation (2) with constants $A = 12.24$ and $B = 1.44$ (BÅTH, 1958) are calculated for shallow and shallow plus intermediate earthquakes. The period considered is from 1897 to 1964. The values of a and b and their standard deviations σ_a and σ_b are calculated by the least squares method.

Tables 1 and 2 give the values of a, σ_a, b, σ_b, mean annual energy released — TE/year — M_1, ΔM_1, M_2, ΔM_2, and the values of M_3 for both analtyical — M_{3A}, ΔM_{3A}, — and graphical — M_{3G} — methods for each subregion for shallow and shallow plus intermediate earthquakes, respectively. Both tables also contain the ratios M_2/M_1, M_3/M_1, the waiting time Tr years, and the difference between M_3 and M_2 for each region. Table 2 also includes the same parameters for the world as a whole for the period 1897 - 1970 (BÅTH, 1973). Results from the graphical method for regions and periods indicated in the appropriate captions are illustrated in Figures 2 to 9.

4. *Results and discussion*

4.1. *General features*

Figures 2-9 and Tables 1 and 2 reveal several general features. The first is that, as the energy release decreases and gets closer to the lower parallel bound, the possibility of having a large earthquake appears to have greater expectation, and vice versa. Thus the lower (upper) parallel bound is the bound of higher (lower) seismic risk for the region, because it is the line of maximum (minimum) storage of energy that may be released. Then the maximum possible energy that may be released in a year is the difference between E_{max} and the level of energy that has already been released during the recent past. The second feature to note is that on graphs like these the energy of large events can dominate because of the logarithmic nature of equation (2). For the circum-Pacific belt, for example, it is clear that the very active period of the first decades of the present century dominates analysis of the seismic behaviour of this part of the world.

A third significant feature is the very good agreement on M_3 as obtained from analytical and graphical methods. This agreement shows that the assumption, that the vertical distance between the two enveloping parallel lines is equivalent to the maximum possible energy that may be released, is a realistic one.

Table 1

Parameters Computed from Shallow Earthquakes

Region	a	b	M_1	TE*/year	M_2	M_{3A}	M_{3G}	M_2/M_1	M_{3A}/M_1	M_{3A}/M_2	M_{3A}-M_2	Tr (years)
(1) South America	5.18 ±.58	0.74 ±.08	6.96 ±.05	5.72	7.99 ±.13	9.05 ±.32	9.01	1.15	1.30	1.13	1.06	27
(2) North America	8.40 ±.62	1.15 ±.07	7.29 ±.04	4.50	7.93 ±.09	8.87 ±.56	9.00	1.09	1.22	1.12	0.94	31
(3) Aleutians, Alaska	5.86 ±.68	0.85 ±.08	6.89 ±.05	3.70	7.86 ±.12	9.03 ±.37	8.97	1.14	1.31	1.15	1.17	21
(4) Japan, Kuril, Kamchatka	8.14 ±.51	1.10 ±.06	7.40 ±.03	8.50	8.11 ±.09	9.19 ±.48	8.91	1.10	1.24	1.13	1.08	16
(5) N. Guinea, Banda Sea, Celebes	9.17 ±.62	1.24 ±.07	7.39 ±.03	6.10	8.02 ±.09	9.04 ±.83	8.92	1.09	1.22	1.13	1.02	19
(6) N. Hebrides, Solomon, N. Guinea	9.27 ±.81	1.27 ±.10	7.27 ±.04	2.99	7.80 ±.06	8.61 ±.84	8.70	1.08	1.18	1.10	0.81	16
(7) N. Zealand, Tonga, Kermadec	6.52 ±.54	0.94 ±.07	6.93 ±.04	2.99	7.80 ±.16	8.96 ±.54	8.93	1.12	1.29	1.15	1.16	41

* Units are 10^{23} erg

Table 2

Parameters Computed from Shallow Plus Intermediate Earthquakes

Region	a	b	M_1	TE*/year	M_2	M_{3A}	M_{3G}	M_2/M_1	M_{3A}/M_1	M_{3A}/M_2	M_{3A}-M_2	Tr (years)
(1) South America	7.54 ±.49	1.04 ±.06	7.24 ±.04	6.44	8.03 ±.12	9.18 ±.54	9.10	1.11	1.27	1.14	1.15	25
(2) North America	8.74 ±.81	1.19 ±.10	7.34 ±.03	4.74	7.94 ±.10	8.86 ±.55	9.00	1.08	1.21	1.12	0.92	33
(3) Aleutians, Alaska	6.73 ±.53	0.95 ±.07	7.07 ±.04	4.06	7.89 ±.11	9.01 ±.41	8.78	1.12	1.27	1.14	1.12	20
(4) Japan, Kuril, Kamchatka	8.42 ±.55	1.12 ±.07	7.52 ±.03	10.43	8.18 ±.07	9.11 ±.42	8.96	1.09	1.21	1.11	0.93	14
(5) N. Guinea, Banda Sea, Celebes	8.23 ±.37	1.10 ±.05	7.48 ±.03	8.62	8.12 ±.06	9.00 ±.36	9.04	1.09	1.20	1.11	0.88	22
(6) N. Hebrides, Solomon, N. Guinea	10.29 ±.44	1.40 ±.06	7.35 ±.04	4.19	7.90 ±.05	8.83 ±0.80	8.83	1.07	1.20	1.12	0.93	21
(7) N. Zealand, Tonga, Kermadec	7.36 ±.31	1.04 ±.04	7.08 ±.03	3.63	7.86 ±.13	9.03 ±.55	8.97	1.11	1.27	1.15	1.17	30
World (1897–1970)	10.44 ±.80	1.29 ±.10	8.09 ±.03	44.00	8.68 ±.04	9.24 ±.53	9.52					18

* Units are 10^{23} erg

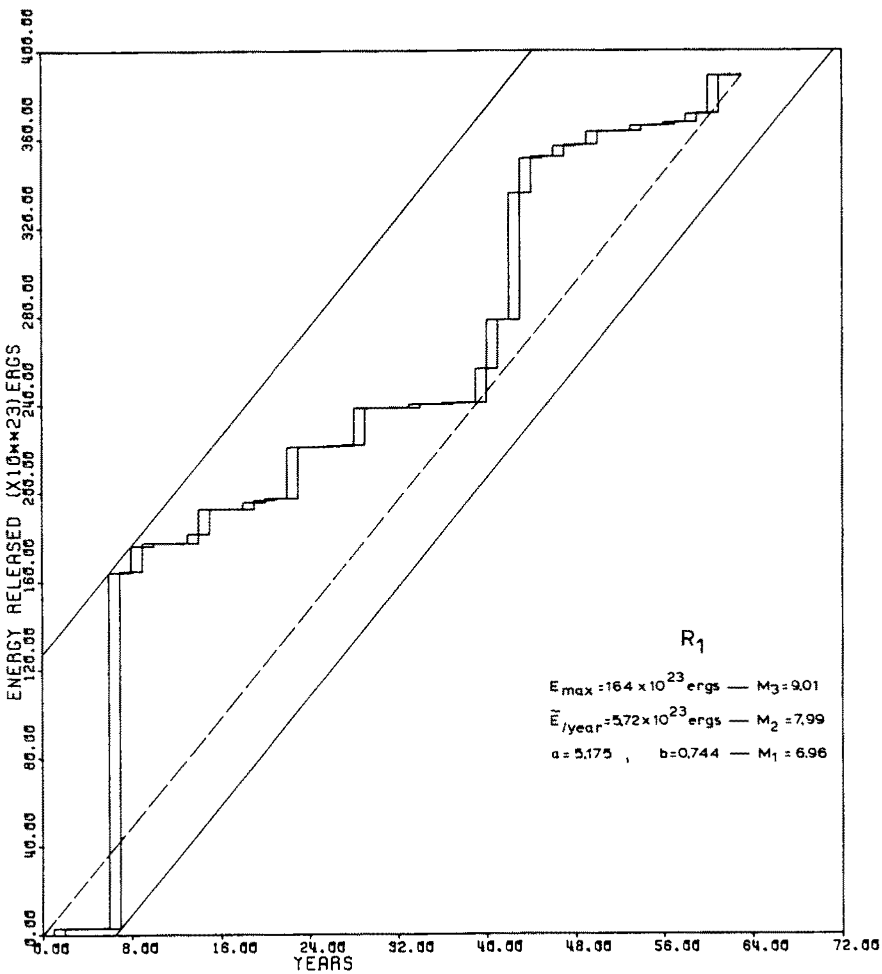

Figure 2. Cumulative energy release as a function of time for circum-Pacific Region 1 (South America) (DUDA, 1965), 1897–1964. The insert shows the values of maximum possible energy release E_{max}, mean annual energy release $\overline{E}$/year, and the values of a and b of the frequency–magnitude formula. The magnitudes that correspond to annual mode (a/b), $\overline{E}$/year, and E_{max} are also listed as M_1, M_2, and M_3, respectively.

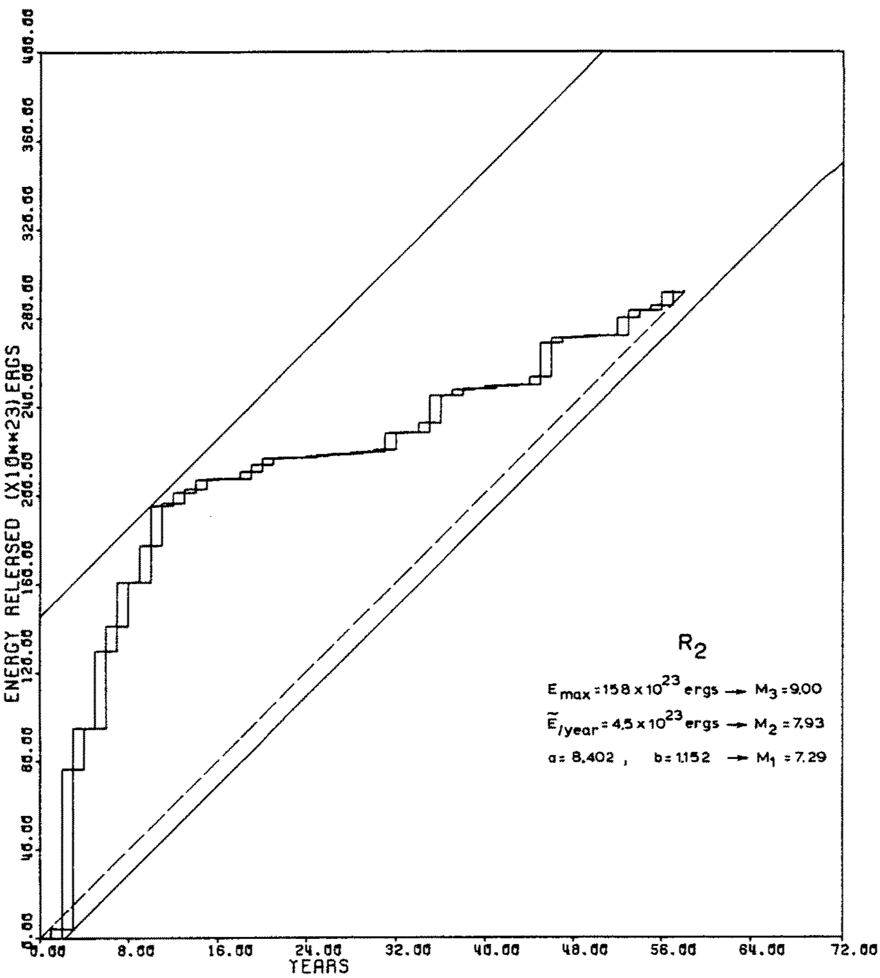

Figure 3. Region 2 (North America). (For explanation see Fig 2.)

Probably the most significant feature is the close relation between M_1, M_2, and M_3. From Table 1 for shallow earthquakes we can derive the following relations:

$$
\begin{aligned}
M_2 &= (1.11 \pm 0.04) M_1, \\
M_3 &= (1.25 \pm 0.05) M_1, \\
M_3 &= (1.13 \pm 0.02) M_2, \\
M_3 - M_2 &= 1.03 \pm 0.13.
\end{aligned}
\tag{23}
$$

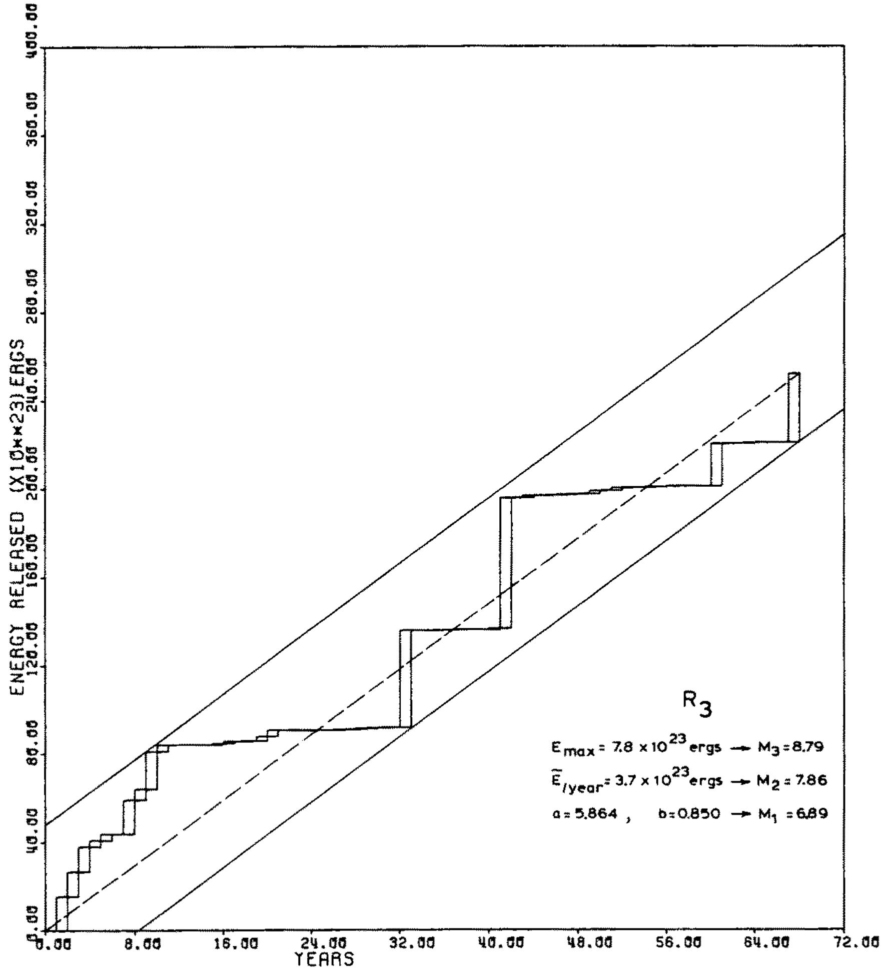

Figure 4. Region 3 (Aleutians, Alaska). (For explanation see Fig 2.)

When the same procedure is applied to the parameters of Table 2 for shallow plus intermediate earthquakes, we find

$$
\begin{aligned}
M_2 &= (1.11 \pm 0.06)M_1, \\
M_3 &= (1.25 \pm 0.07)M_1, \\
M_3 &= (1.13 \pm 0.03)M_2, \\
M_3 - M_2 &= 1.04 \pm 0.13.
\end{aligned}
\tag{24}
$$

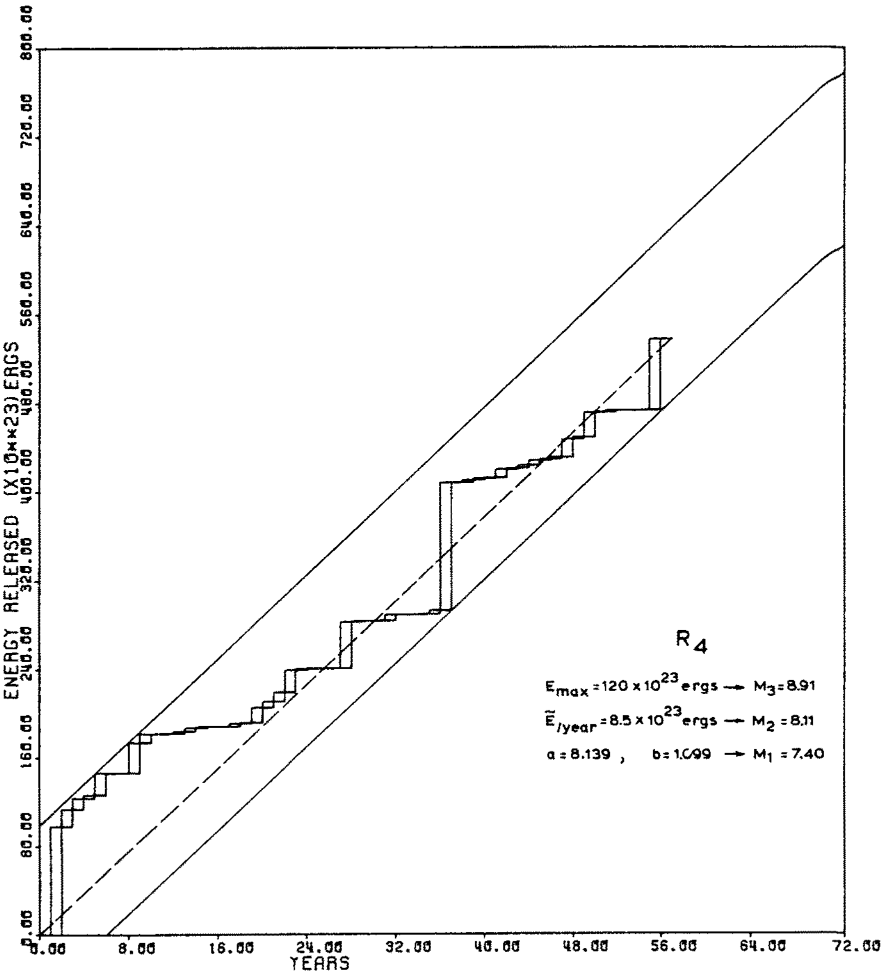

Figure 5. Region 4 (Japan, Kuril, Kamchatka). (For explanation see Fig. 2.)

Relations (23) and (24) are similar. This may be because the same mechanisms characterize both shallow and intermediate earthquakes in the depth range of 0–400 km (Bäth and Duda, 1963). However, because Duda's subdivision is based on the distribution and number of shallow earthquakes rather than on tectonic evidence, relations (23) and (24) seem to be valid for tectonically very different regions and for a wide range of magnitudes.

Considering the ease with which M_1 and M_3 may thus be derived for a region, these equations will be of great assistance for regional seismic

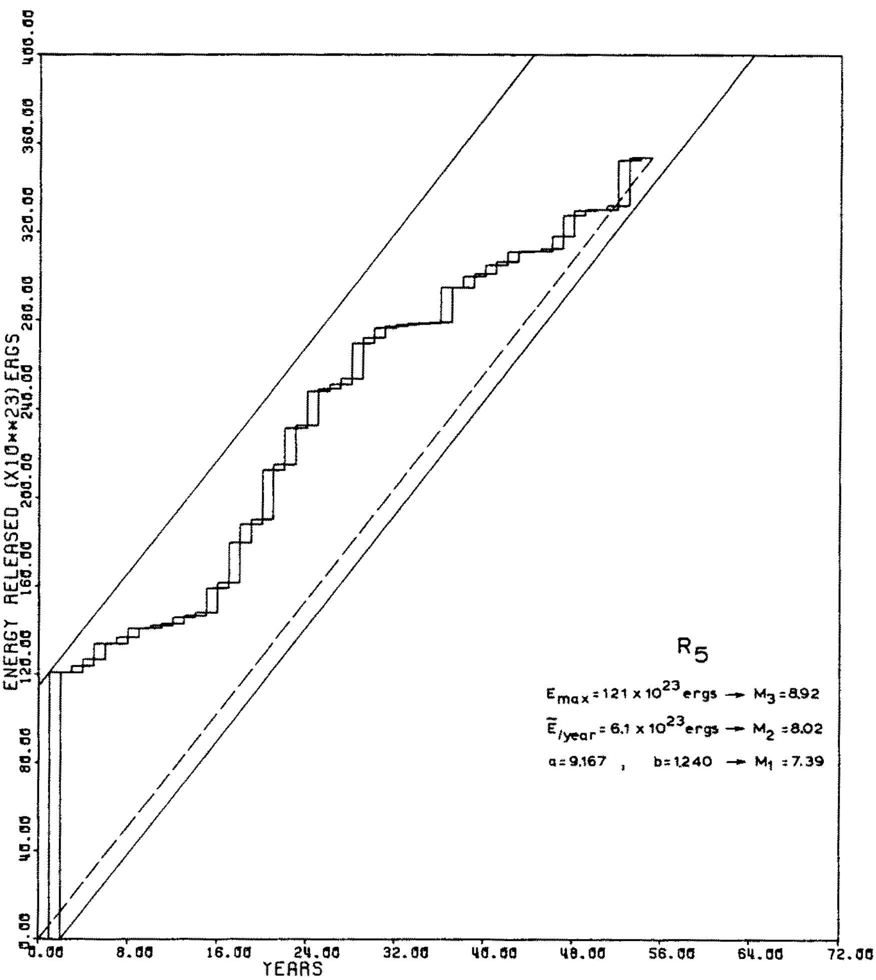

Figure 6. Region 5 (New Guinea, Banda Sea, Celebes, Moluccas, Philippines). (For explanation see Fig. 2.)

risk considerations, particularly if, as seems likely, they have a universal character.

4.2 Regional features

(i) Circum-Pacific belt: From Figures 2–8 a general pattern of decreasing activity is apparent for all the circum-Pacific belt subregions after the high seismic activity during the first two decades of this century. The observation

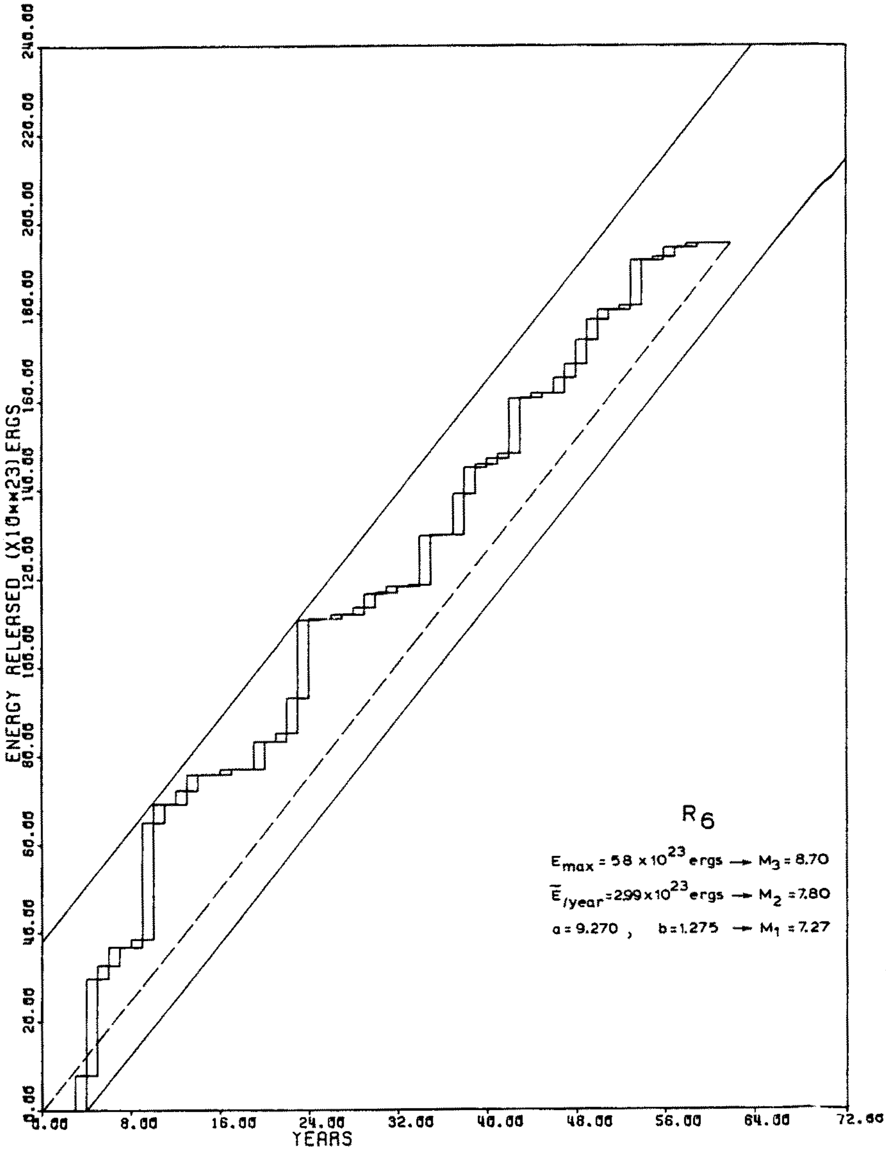

Figure 7. Region 6 (New Hebrides, Solomon's, New Guinea). (For explanation see Fig. 2.)

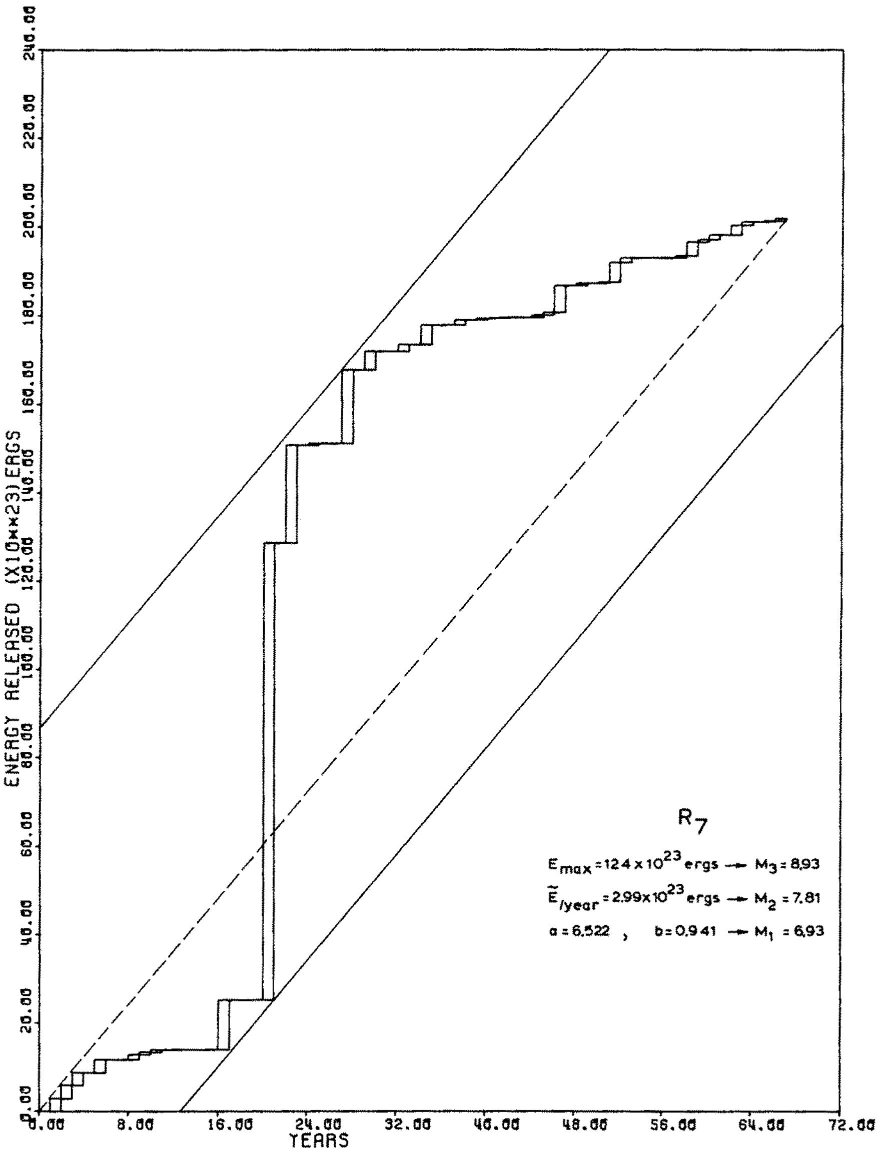

Figure 8. Region 7 (New Zealand, Tonga, Kermadec). (For explanation see Fig. 2.)

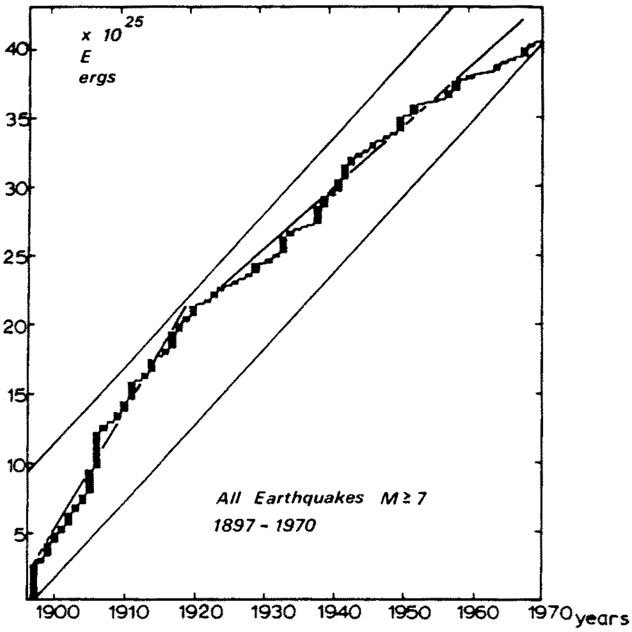

Figure 9. Cumulative energy release as a function of time for the world, over the period 1897–1970 (after BÅTH, 1973).

that in 1965, in almost all cases, the level of energy released was close to the lower bound suggests that a period of increasing seismic activity may be about to start. This, in fact, seems to be the case. Looking back to the international data file, for the period 1965–1974 we can see that Region 1 has experienced 27 quakes with magnitude range 7.0–8.1; Region 2, 13 quakes, magnitude 7.0–8.5; Region 3, 13 quakes, magnitude 7.0–7.9; Region 4, 27 quakes, magnitude 7.0–8.1; Region 5, 35 quakes, magnitude 7.0–8.1; Region 6, 49 quakes, magnitude 7.0–8.1; Region 7, 12 quakes, magnitude 7.0–7.9. The waiting times *Tr* years for large shallow earthquakes in Regions 1 to 7 are 27, 31, 21, 16, 19, 16, and 41 years, respectively (see Table 1). The two most active regions are situated diagonally opposite each other. These are the northwestern part (Region 4) and southeastern part (Region 1) of the circum-Pacific belt, with $M_3 = 9.2\ (\pm 0.5)$ and $9.1\ (\pm 0.3)$, respectively. Region 4 also has the shorter waiting time, which means that a period of 16 years without any large earthquake is enough to accumulate energy for an earthquake with magnitude as high as 9.2.

(ii) World: Table 2 also contains the parameters for shallow and intermediate earthquakes for the world as a whole. Since these earthquakes occurred in a broad variety of seismotectonic environments, the parameters b and M_1 are only of limited interest. M_{3G}, however, which does not depend on the value of b, is significant as an indication of the global upper bound for

earthquake magnitudes around 9.5, M_{3A} indicating a magnitude around 9.24. It is apparent from Table 2 that estimates of M_{3A}, for example for South America and Japan, are little different from those for the world as a whole. They certainly should not be taken as statistically significantly different, as their uncertainties make clear. These values are consistent with the notion that the value of M_3 found for the world as a whole may occur in some of its subregions in the circum-Pacific.

5. Conclusions

The analytical method described herein uses a simple linear frequency–magnitude law and demonstrates the necessary existence of an upper limit to maximum magnitude earthquakes, if this distribution is to be compatible with maintenance of finite rates of strain energy release. This result need not be mathematically true for other frequency magnitude distributions, although as an assumption it would be physically realistic. From both the analytical and the graphical methods of relating seismicity to strain energy release it is possible to estimate the size of such an upper limit. Thus, preferred statistical models of earthquake magnitude occurrence should include this upper limit as an unknown parameter, in order to simulate the physical reality. The advantage of the methods described above is the ease with which the size of the upper magnitude limit M_3 may be obtained from the known M_1 and M_2 through relation (14) or (16).

The two methods, when tested in the circum-Pacific belt, using Duda's catalogue and regionalisation, show a very good agreement between M_3 as obtained from the analytical and graphical methods.

From the regional features it is known that the two most active regions in the circum-Pacific belt are Region 4 (Japan, Kuril, Kamchatka) and Region 1 (South America). These regions are situated diagonally opposite each other, and they have upper bounds for shallow earthquakes of $M_3 = 9.2$ (± 0.5) and 9.1 (± 0.3), respectively, compatible with a global upper bound to all earthquake occurrence $M_3 = 9.2$ (± 0.5). The Japan–Kuril–Kamchatka zone also has the shortest waiting time: 16 years. In almost all the circum-Pacific belt a general pattern of decreasing activity is observed, approaching 1964. The historical cumulative energy release was observed in all cases nearer to the lower than the higher strain energy release bound (maximum energy storage), compatible with a period of increasing seismic activity starting after 1965.

For the world as a whole, values of M_3 from around 9.2 to 9.5 are found from the two methods to indicate the global upper bound for earthquake magnitudes compatible with finite rates of observed strain energy release. This global upper bound is compatible with its occurrence in several subregions of the circum-Pacific.

The empirically obtained sets of relations (23) and (24) are almost identical, which may be explained if the shallow and intermediate depth earthquakes arise from the same seismicity process, with similarity of mechanism in the focal depth range 0–400 km (BÅTH and DUDA, 1963). If the close empirical relations between the annual largest earthquake M_1, the magnitude equivalent to the average annual strain energy release M_2, and the upper bound to earthquake magnitude M_3, which have been derived from tectonically dissimilar regions, have a universal character, then equation (23) or (24) will be of great assistance in regional seismic risk considerations. *The upper bound to earthquake magnitude occurrence and the magnitude equivalent of the annual rate of strain energy release differ by one magnitude unit.*

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REFERENCES

ABE K., and KANAMORI, H. (1980), *Magnitudes of great shallow earthquakes from 1953 to 1977,* Tectonophysics *62,* 191–203.

AMBRASEYS, N. N. (1971), *Value of historical records of earthquakes,* Nature *232,* 375–379.

BÅTH, M., *The energies of seismic body waves and surface waves.* (In, H. Benioff, M. Ewing, B. F. Howell, Jr., and F. Press, eds, *Contributions in Geophysics.*) London: Pergamon, 1958, 1.16.

BÅTH, M. *Introduction to Seismology.* (Birkhauser Verlag, Basel 1973).

BÅTH, M. (1981), *Earthquake magnitude—Recent research and current trends,* Earth-Sci. Rev. *17,* 315–398.

BÅTH, M., and DUDA, S. J. (1963), *Strain release in relation to focal depth,* Geofis. Pura & Appl. *56,* 93–100.

CORNELL, C. A., and VANMARCKE, E. H. (1969), *The major influences on seismic risk,* 4th World Conf. on Earthquake Engineering, Chile.

COSENTINO, P., FICARRA, V., and LUZIO, D. (1977), *Truncated exponential frequency–magnitude relationship in earthquake statistics,* Bull. Seismol. Soc. Am. *67,* 1615–1623.

DEWEY, J. W. (1976), *Seismicity of Northern Anatolia,* Bull. Seismol. Soc. Am. *66,* 843–868.

DUDA, S. J. (1965), *Secular seismic energy release in the circum-Pacific belt,* Tectonophysics *2,* 409–452.

GALANOPOULOS, A. G. (1972), *Annual and maximum possible strain accumulation in the major area of Greece,* Ann. Geol. D. pays Hellen *24,* 467–480.

GUMBEL, E. J. *Statistics of Extremes.* New York: Columbia University Press, 1958.

GUTENBERG, B., and RICHTER, C. F. (1944), *Frequency of earthquakes in California*, Bull. Seismol. Soc. Am. *34*, 185–188.

HÉDERVÁRI, P., and PAPP, Z. (1977), *Seismicity maps of the New Guinea–Solomon Islands region*, Tectonophysics *42*, 261–281.

HÉDERVÁRI, P., and PAPP, Z. (1981), *Seismicity maps of the Indonesian region*, Tectonophysics *76*, 131–148.

KANAMORI, H. (1977), *The energy release in great earthquakes*, J. Geophys. Res. *82*, 2981–2987.

KNOPOFF, L., and KAGAN, Y. (1978), *Analysis of the theory of extremes as applied to earthquake problems*, J. Geophys. Res. *82*, 5647–5675.

LEE, W. H. K., WU, F. T., and JACOBSON, C. (1976), *A catalog of historical earthquakes in China compiled from recent Chinese publications*, Bull. Seismol. Soc. Am. *66*, 2003–2016.

LOMNITZ, C. *Global Tectonics and Earthquake Risk (Developments in Geotectonics 5)*. Amsterdam: Elsevier, 1974.

MAKROPOULOS, K. C. (1978), *The statistics of large earthquake magnitude and an evaluation of Greek seismicity*. PhD Thesis, Edinburgh University.

MERZ, H., and CORNELL, C. A. (1973), *Seismic risk analysis based on a quadratic magnitude–frequency law*, Bull. Seismol. Soc. Am. *63*, 1999–2006.

SOYSAL, H., KOLCAK, D., and SIPAHIOGLU, S. *Some aspects of the North Anatolian Fault Zone derived from the comparison of its instrumental data with historical information. Proc. Internat. Symposium on Earthquake Prediction in the North Anatolian Fault Zone*, Istanbul, Turkey, 1980 March 31–April 5. (In, A. Mete Isikara and Andreas Vogel, eds, *Multidisciplinary Approach to Earthquake Prediction.*) Wiesbaden: Braunschweig, 1982, pp. 223–238.

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